



Topological Quantum Chemistry: The Topological Materials Database

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Introduction

Topological materials: second revolution in quantum mechanics



Classification as a function of conductivity

Semiconductors







Metals





Introduction

What's a topological insulator?

New classification of matter









Insulator

Metal





Non-topological (trivial) insulator





Quantum Hall Effect : bulk-boundary correspondence



K. v. Klitzinget al. Phys. Rev. Lett. 45, 494 (1980)



Introduction

Berry phase





Insulators





Introduction





Introduction

Insulators

Band insulators







Topology and symmetry

Quantum Hall effect (edge states)



K. v. Klitzinget 1980, Haldane 1988, Kane and Mele 2005, Liang Fu 2011



Topological classification before crystalline symmetries

 $\Theta H(\mathbf{k})\Theta^{-1} = +H(-\mathbf{k}); \quad \Theta^2 = \pm 1 \longrightarrow \text{Time Reversal}$ $\Xi H(\mathbf{k})\Xi^{-1} = -H(-\mathbf{k}); \quad \Xi^2 = \pm 1 \longrightarrow \text{Particle - Hole}$ $\Pi H(\mathbf{k})\Pi^{-1} = -H(\mathbf{k}); \quad \Pi \propto \Theta \Xi \longrightarrow \text{Chiral symmetry}$

	Symn	netry	d									
AZ	Θ	Ξ	Π	1	2	3	4	5	6	7	8	
Α	0	0	0	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	
AIII	0	0	1	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0	
AI	1	0	0	0	0	0	\mathbb{Z}	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	
BDI	1	1	1	\mathbb{Z}	0	0	0	\mathbb{Z}	0	\mathbb{Z}_2	\mathbb{Z}_2	
D	0	1	0	\mathbb{Z}_2	\mathbb{Z}	0	0	0	\mathbb{Z}	0	\mathbb{Z}_2	
DIII	-1	1	1	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0	\mathbb{Z}	0	
AII	-1	0	0	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0	\mathbb{Z}	
CII	-1	-1	1	\mathbb{Z}	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0	
\mathbf{C}	0	-1	0	0	\mathbb{Z}	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	
CI	1	-1	1	0	0	\mathbb{Z}	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	
	AZ A AIII AI BDI D DIII AII CII C CI	$\begin{array}{c c} Symm\\ AZ & \Theta\\ \hline A & 0\\ AIII & 0\\ \hline AI & 1\\ BDI & 1\\ D & 0\\ DIII & -1\\ AII & -1\\ CII & -1\\ CII & -1\\ C & 0\\ CI & 1\\ \end{array}$	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c c c c c c c c c c c c c } Symmetry & AZ & \Theta & \Xi & \Pi \\ \hline A & 0 & 0 & 0 \\ \hline A & 0 & 0 & 0 \\ \hline A & 0 & 0 & 1 \\ \hline A & 0 & 0 & 1 \\ \hline A & 1 & 1 & 0 & 0 \\ \hline B & 1 & 1 & 1 & 1 \\ \hline D & 0 & 1 & 0 \\ \hline D & 0 & 1 & 0 \\ \hline D & 0 & 1 & 0 \\ \hline D & 1 & -1 & 1 \\ \hline D & 0 & -1 & 0 \\ \hline C & 0 & -1 & 0 \\ \hline C & 1 & -1 & 1 \\ \end{array}$	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Symmetry II 1 2 3 4 AZ Θ Ξ II 1 2 3 4 A 0 0 0 0 Z 0 Z 0 Z AIII 0 0 1 Z 0 Z 0 Z 0 AI 1 0 0 1 Z 0 0 Z 0 AI 1 0 0 0 0 0 0 Z 0 BDI 1 1 1 Z 0 0 0 Z D 0 1 0 0 Z_2 Z 0 0 DIIII -1 1 1 Z_2 Z_2 Z 0 AII -1 0 0 Z_2 Z_2 Z 0 CII -1 -1 1 0 0 Z_2 2 Z CI 1 -1 1 0 0 Z_2 <td>Symmetry d AZ Θ Ξ Π 1 2 3 4 5 A 0 0 0 0 Z 0 Z 0 AIII 0 0 1 Z 0 Z 0 Z 0 AI 1 0 0 0 0 0 Z 0 Z AI 1 0 0 0 0 Z 0 Z BDI 1 1 1 Z 0 0 Z 0 Z D 0 1 0 Z_2 Z_2 Z 0 0 DIII -1 1 1 Z_2 Z_2 Z 0 0 Z Z<td>$\begin{array}{c c c c c c c c c c c c c c c c c c c$</td><td>$\begin{array}{c ccccccccccccccccccccccccccccccccccc$</td></td>	Symmetry d AZ Θ Ξ Π 1 2 3 4 5 A 0 0 0 0 Z 0 Z 0 AIII 0 0 1 Z 0 Z 0 Z 0 AI 1 0 0 0 0 0 Z 0 Z AI 1 0 0 0 0 Z 0 Z BDI 1 1 1 Z 0 0 Z 0 Z D 0 1 0 Z_2 Z_2 Z 0 0 DIII -1 1 1 Z_2 Z_2 Z 0 0 Z <td>$\begin{array}{c c c c c c c c c c c c c c c c c c c$</td> <td>$\begin{array}{c ccccccccccccccccccccccccccccccccccc$</td>	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	

Altland-Zirnbauer Random Matrix Classes

Ryu, Schnyder, Furusaki, Ludwig New J. Phys. 12, 065010 (2010)



Introduction



Unstoppable currents, constant conductivity, spin-momentum locking (in 2D and 3D) Independent of size, weak disorder and temperature



Photovoltaics Catalysis

Energy conversion Materials for future Quantum Technologies

Environmentally friendly technology

Quantum Anomalous Hall Topological Surface States

Chirality + topology Dark Matter detectors

Dark Matter detectors Quantum gates: Braiding



Non-abelian anyons Fractional TIs



Topological classification before crystalline symmetries

Example: Z₂ invariant in 2D and 3D Topological Insulators (protected by TRS) Band theory (Fu, Kane & Male (2007), Moore and Balents (2007), Roy (2007)



$$W_{nm}(k_x, k_y) = \overline{\exp}\left[\int_0^{2\pi} dk_z \langle u_m(\vec{k}) | \partial_{k_z} | u_n(\vec{k}) \rangle\right]$$

unitary operator in filled
band subspace

Define 'Wilson loop Hamiltonian'
$$W(k_x,k_y) = \exp[-\mathrm{i} H_\mathrm{W}(k_x,k_y)]$$
 $\pi exp[-\mathrm{i} H_\mathrm{W}(k_x,k_y)]$



Band inversion criteria



$$(-1)^{\nu} = \Pi \delta i = \pm 1$$

 $\nu = \begin{cases} -1 \text{ topological insulator} \\ 1 \text{ trivial insulator} \end{cases}$



Band inversion criteria

Example: Z₂ invariant in 2D and 3D Topological Insulators (protected by TRS) Band theory (Fu, Kane & Male (2007), Moore and Balents (2007), Roy (2007)



 $(-1)^{\nu} = \Pi \delta i = \pm 1$



Topological materials





Topological materials





Topological materials





Crystal Structure





Ingredients:

- unit lattice translations (Z³)
- point group operations (rotations, reflections)
- non-symmorphic (screw, glide)
- orbitals
- atoms in some lattice positions





Nature (2017)

FRESH TWIST ON TOPOLOGY





$$\left|\varphi(R)\right\rangle = \oint_{BZ} \frac{dk}{2\pi} e^{-ikR} \left|\psi_{k}\right\rangle = \oint_{BZ} \frac{dk}{2\pi} e^{-ik(R-r)} \left|u_{k}\right\rangle$$















Bloch states $\psi_k(r) = e^{ikr}u_k(r)$ are defined for periodic boundary conditions

Atomic limit



 $\mathsf{R}_{\langle r \rangle}$



orbital + atomic site + lattice (irrep + wyckoff position + space group)



atomic limit = EBR

An EBR describes a set of Wannierizable bands

Zak, "Symmetry specification of bands in solids," Physical Review Letters 45, 1025 (1980), Band representations and symmetry types of bands in solids," Physical Review B 23, 2824 (1981), Band representations of space groups," Physical Review B 26, 3010 (1982).

Elementary band representations (EBRs)



Elementary Band Representations

B. Bradlyn et al. Nature 547 (2017), M.G. Vergniory et al. Nature 566 (2019), M. G. Vergniory et al. Science (2022)

Compatibility Relations

Topological Quantum Chemistry



Nature (2017)

FRESH TWIST ON







Nature (2017)

RhSi



RhSi









Obstructed Wannier Charge Center



SSH chain in P-1





CdSb in Pbca (No. 61)

Cleavage cuts the OWCC



<u>2</u>76







EBR induced at 2b = TOP_BANDS + EBR induced at 1a (OAL)

Fragile: EBR_F=EBR1-EBR2

Phys. Rev. Lett. 120, 266401 (2018)





Phys. Rev. Lett. 120, 266401 (2018)

Topological Fragile Phase



EBR induced at 2b = TOP_BANDS + EBR induced at 1a (OAL)

Fragile: EBR_F=EBR1-EBR2

Phys. Rev. Lett. 120, 266401 (2018)









2019 TQC (2 years) ~10k topological materials









All gaps in the energy spectrum





Magnetic co-representations



Magnetic codes



Magnetic Topological Classification



BCS Applications Implemented for MTQC										
Application	Contents	Description								
MKVEC	Momentum stars	SA D1								
	of the MSGs									
Corepresentations	Small and full	SA D2								
	magnetic (co)reps									
MCOMPREL	Compatibility relations	SA D3								
	in the MSGs									
CorepresentationsPG	Magnetic site-symmetry	SA E1								
	group (co)reps									
MSITESYM	Magnetic small	SA E2								
	(co)reps at one \mathbf{k} point									
	induced from a site ${\bf q}$									
MBANDREP	MEBRs of the MSGs	SA E3								



SSGs (1651)	Type-I MSGs (230)	Type-II SGs (230)	Type-III MSGs (674)	Type-IV MSGs (517)
(Co)reps	1	1	1	1
Compatibility rel.	1	1	1	1
EBRs	1	1	1	1
Enforced SMs	1	1	1	1
SI group	1	1	1	1
SI formulas	1	1	1	1
Fragile criteria		1		
Stable invariants	*	1	*	*
Boundary states	*	1	*	*
$SI \rightarrow invariants$	1	1	1	1



First Principles Calculations: 103 materials with non trivial topology

Categories	Properties	Materials
I-A	Non-collinear Manganese compounds	Mn ₃ GaC, Mn ₃ ZnC, Mn ₃ CuN, Mn ₃ Sn, Mn ₃ Ge, Mn ₃ Ir, Mn ₃ Pt, Mn ₅ Si ₃
I-B	Actinide Intermetallic	UNiGa ₅ , UPtGa ₅ , NpRhGa ₅ , NpNiGa ₅
I-C	Rare earth intermetallic	NdCo ₂ , TbCo ₂ , NpCo ₂ , PrAg DyCu, NdZn, TbMg, NdMg,
		Nd_5Si_4 , Nd_5Ge_4 , Ho_2RhIn_8 , Er_2CoGa_8 , Nd_2RhIn_8 , Tm_2CoGa_8 ,
		$Ho_2RhIn_8, DyCo_2Ga_8, TbCo_2Ga_8, Er_2Ni_2In, CeRu_2Al_{10}, Nd_3Ru_4Al_{12},$
		$Pr_3Ru_4Al_{12}$, $ScMn_6Ge_6$, YFe_4Ge_4 , $LuFe_4Ge_4$, $CeCoGe_3$
II-A	Metallic Iron pnictides	$LaFeAsO, CaFe_2As_2, EuFe_2As_2, BaFe_2As_2, Fe_2As, CaFe_4As_3,$
		LaCrAsO, Cr_2As , $CrAs$, CrN
II-B	Semiconducting manganese pnictides	$BaMn_2As_2 BaMn_2Bi_2$, $CaMnBi_2$, $SrMnBi_2$, $CaMn_2Sb_2$, $CuMnAs$,
		CuMnSb, Mn ₂ As
II-C	Rare earth intermetallic compounds with	$PrNi_2Si_2$, $YbCo_2Si_2$, $DyCo_2Si_2$, $PrCo_2P_2$, $CeCo_2P_2$, $NdCo_2P_2$,
	the composition 1:2:2	$DyCu_2Si_2, CeRh_2Si_2, UAu_2Si_2, U_2Pd_2Sn, U_2Pd_2In, U_2Ni_2Sn,$
		U_2Ni_2In, U_2Rh_2Sn
II-D	Rare earth ternary compounds of the	CeMgPb, PrMgPb, NdMgPb, TmMgPb
	composition 1:1:1	
III-A	Semiconducting Actinides/Rare earth	HoP, UP, UP ₂ , UAs, NpS, NpSe, NpTe, NpSb, NpBi, U_3As_4 , U_3P_4
	Pnictides	
III-B	Metallic oxides	Ag ₂ NiO ₂ , AgNiO ₂ , Ca ₃ Ru ₂ O ₇ , Double perovskite Sr ₃ CoIrO ₆
III-C	Metal to insulator transition compounds	NiS_2 , $Sr_2Mn_3As_2O_2$
III-D	Semiconducting and insulating oxides,	$LuFeO_3$, $PdNiO_3$, $ErVO_3$, $DyVO_3$, $MnGeO_3$, $Tm_2Mn_2O_7$, $Yb_2Sn_2O_7$,
	borates, hydroxides, silicates, phosphate	$Tb_2Sn_2O_7$, $Ho_2Ru_2O_7$, $Er_2Ti_2O_7$, $Tb_2Ti_2O_7$, $Cd_2Os_2O_7$, $Ho_2Ru_2O_7$,
		Cr_2ReO6 , $NiCr_2O_4$, MnV_2O_4 , Co_2SiO_4 , Fe_2SiO_4 , $PrFe_3(BO_3)_4$,
		$\mathrm{KCo}_{4}(\mathrm{PO}_{4})_{3}, \mathrm{CoPS}_{3}, \mathrm{SrMn}(\mathrm{VO}_{4})(\mathrm{OH}), \mathrm{Ba}_{5}\mathrm{Co}_{5}\mathrm{ClO}_{13}, \mathrm{FeI}_{2}$









Topological Materials Database

https://www.topologicalquantumchemistry.org

Topological Materials Database 24905 Materials: 4339 Topological Insulators, 10061 Semi-Metals											Q Searct	ı 🛈 Ab	out 🛞 (
Compound Contains Only these elements Exclude											ICSD Nun	nber					
e.g. Bi	e.g. Bi1 Se2 Ge					e	eg. 01 N - or -			eg. 123456			Search				
• Show	Advanced	Search															
1																	2
н													He				
Li	Be		B C N O F										¹⁰ Ne				
Na	¹² Mg	1									¹³	¹⁴ Si	15 P	¹⁶ S	17 CI	Ar	
¹⁹ K	Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	Fe	27 Co	28 Ni	29 Cu	³⁰ Zn	an Ga	Ge	aa As	³⁴ Se	as Br	²⁶ Kr
37 Rb	³⁸ Sr	29 Y	40 Zr	41 Nb	42 Mo	43 Tc	Ru	⁴⁵ Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	Te	Si I	⁵⁴ Xe
55 Cs	Ba	57 La	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	⁸⁰ Hg	81 TI	82 Pb	Bi	84 Po	es At	Rn





5,3

UPV EHU

FCT/ZTF



Crystallography Online: Workshop on the use of the structural and magnetic tools of the Bilbao Crystallographic Server September 2021, Leioa (Spain)

Forthcoming schools and workshops

News:

New Article in Nature
 10/2020: Xu et al. "High-throughput
 calculations of magnetic topological
 materials" Nature (2020) 586, 702-707.
 New programs: MBANDREP,
 COREPRESENTATIONS,
 COREPRESENTATIONS,

MCOMPREL, MSITESYM, MKVEC, Check Topological Magnetic Mat 10/2020: new tools in the sections "Magnetic Symmetry and Applications" and "Representations and Applications". More info

 New section: TOPOLOGICAL QUANTUM CHEMISTRY 10/2020: tools for the identification of the







Public codes





Bilbao Crystallographic Server → Check Topological Mat **Check Topological Mat Check Topological Mat.** Upload your traces.txt file (see the help in the column on the left). Browse No file selected Given a file that contain the eigenvalues at each maximal k-vec of a space group, the program gives the set of irreducible representations at each maximal k-vec (time-reversal is assumed). Then, using the compatibility Show relations and the set of Elementary Band Representations (EBRs), it checks whether the set of bands can be put as linear combinations of EBRs. This (self-explanatory) file shows the format of the file to be uploaded in the menu on the right: File_Description https://irrep.dipc.org You can download examples of input files here: Example_Ag1Ge1Li2 Example_Ag1O2Sc1 Example_B2Ca3Ni7 Example_of_Bad_File Example Ba3Ca1O9Ru2 You can generate the "trace.txt" file in your own computer using VASP and this program (fc **IRREP** 🕋 » IrRep vasp2trace Read the "README.pdf" file for help on the use of vasp2trace.

Alternatively, you can use the irvsp package.

If you are using "Check Topological Mat." and/or "vasp2trace" programs in the preparation cite this reference:

M.G. Vergniory, L. Elcoro, C. Felser, N. Regnault, B.A. Bernevig, Z. Wang Nature(2019) 56 doi:10.1038/s41586-019-0954-4

Link to the catalogue of topological materials,

www.topologicalquantumchemistry.com





is a code to calculate symmetry eigenvalues of electronicBloch states in crystalline solids and the irreducible representations under which they transform. It can receive as input bandstructures computed with VASP^G, Abinit^G, Quantum Espresso^G or any code with an interface to Wannier90

Help

Characteristics

- Any space group It can be applied to bandstructures of crystals in any of the 230 space groups preserving time-reversal symmetry.
- spinful or spinless It includes both, single (spinless) and double-valued (spinful) groups. Also, it accepts calculations with spin-orbit coupling corrections.
- Any unit cell Bandstructures calculated with any choice of the unit cell are welcome; primitive, convenctional....
- A trace.txt file that can be passed directly to CheckTopologicalMat^C is generated.
- Adding interfaces to other DFT codes is easy. You are welcome!

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6d 🍀

2h

 \mathbf{a}_2

Topological Photonic Crystals in 2D





Haldane and Raghu, PRL (2008), PRA (2008)

Blanco de Paz, PRR 1, 032005(R), J. Phys.: Condens. Matter 34 314002, Advanced Quantum Technologies 3 (2), 1900117, Devescovi Nat Commun 12, 7330 (2021)



MPB computed bands, GTPack computed representations

$$\nabla \times \left(\frac{1}{\epsilon(\mathbf{r})} \times \mathbf{H}(\mathbf{r})\right) = \left(\frac{\omega}{c}\right)^2 \mathbf{H}(\mathbf{r})$$









MPB computed bands, GTPack computed representations









MPB computed bands, GTPack computed representations







MPB computed bands, GTPack computed representations

Wilson loops of Fragile phase







Band Topology

- We can claim we have understood band topology very deeply
- Topology was unknown but not a rarity
- High throughput searches for magnetic and non-magnetic materials



Nature Reviews Materials, 1-21 (2021)