

Topological Phases of the interacting SSH model



Presenter: Anouar Moustaj

Supervisor: Cristiane Morais Smith



In collaboration with

Emanuele di Salvo, Chen Xu,

Lars Fritz, Andrew Mitchell, Dirk Schuricht

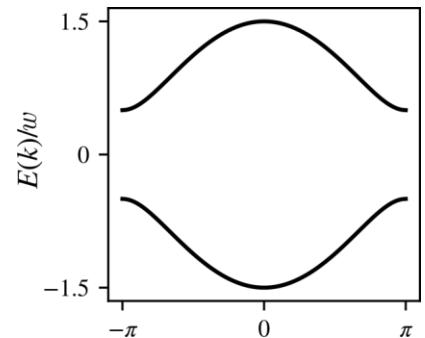
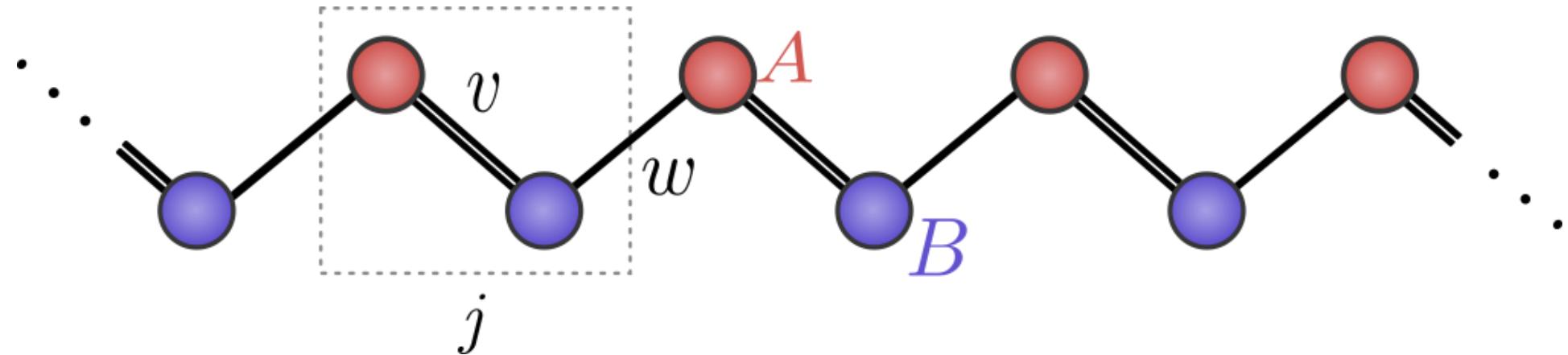


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SSH Model



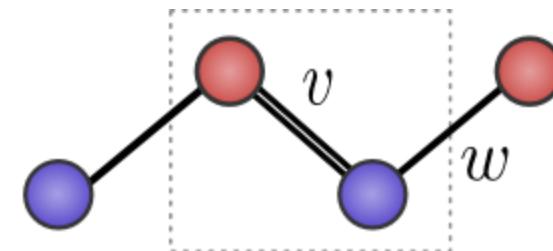
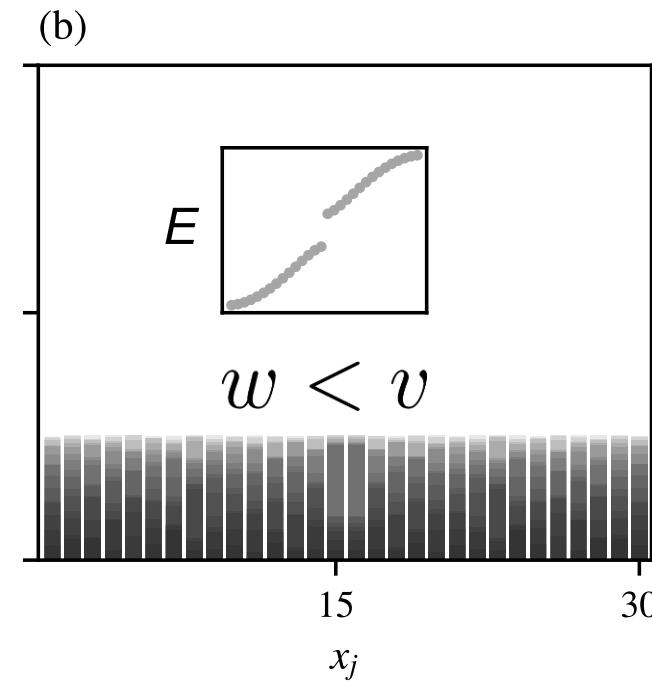
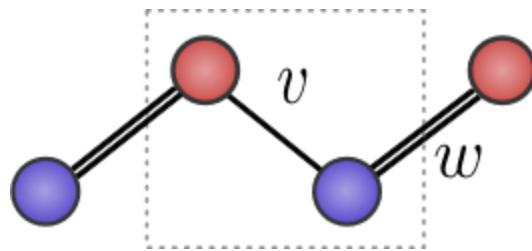
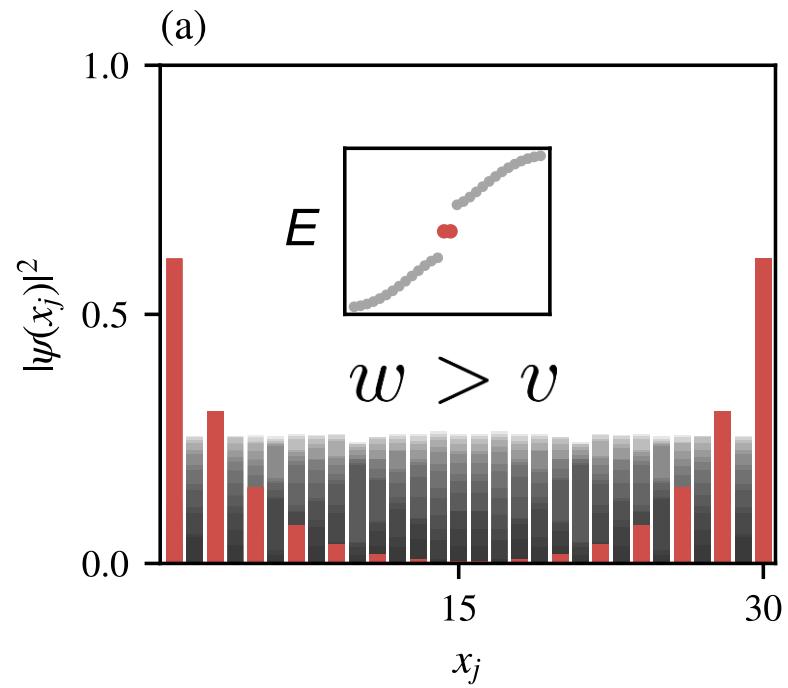
$$\hat{H}_0 = v \sum_j c_{j,A}^\dagger c_{j,B} + w \sum_j c_{j,B}^\dagger c_{j+1,A} + \text{h.c.}$$

Topology - Chiral symmetry

$$[H(k), \sigma_z]_+ = 0$$

$$\nu = \int_{-\pi}^{\pi} \frac{dk}{4\pi i} \text{Tr} \left[\sigma_z H^{-1}(k) \frac{dH(k)}{dk} \right] = \begin{cases} 1, & \text{if } w > v \\ 0, & \text{if } w < v \end{cases}$$

Bulk-Boundary Correspondence



Physical Observable - Polarization

$$P = e\langle \hat{X} \rangle$$

Modern Theory

Berry phase


$$P = e \frac{\phi}{2\pi} \mod e$$

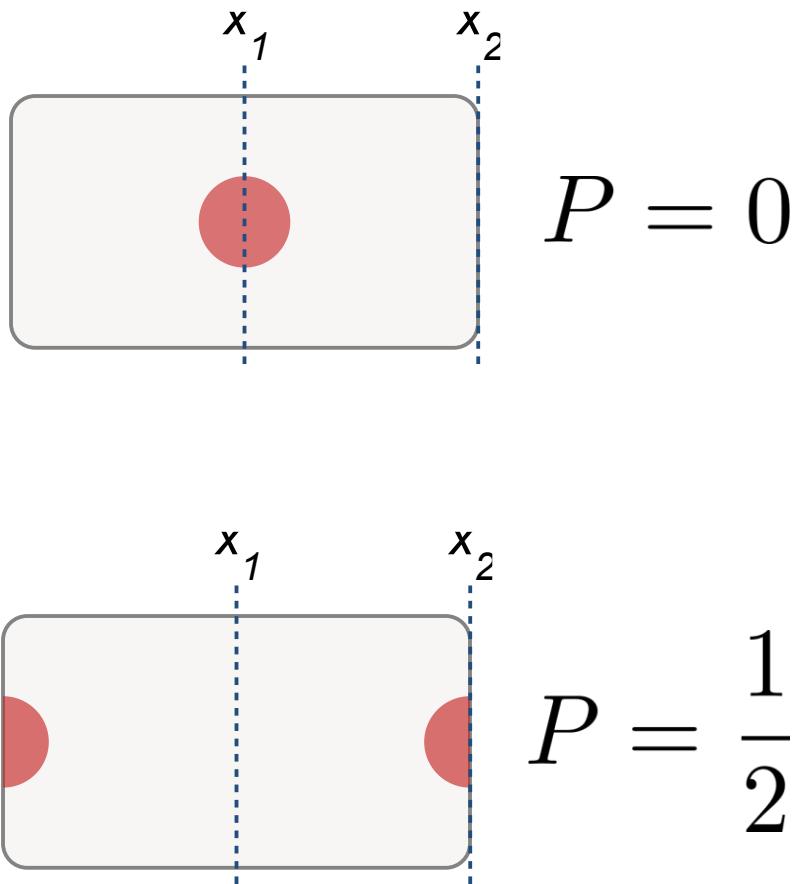
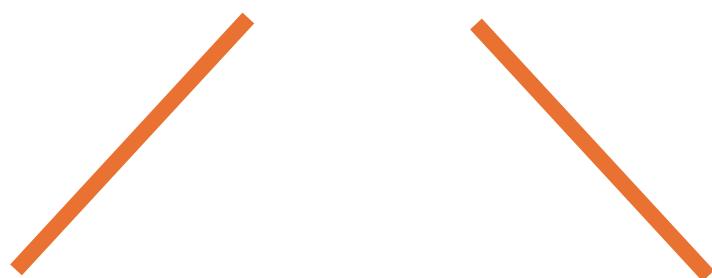
Polarization quantization

Inversion symmetry

$$\phi = 0, \pi$$



$$P = 0, \frac{1}{2}$$



Polarization – topological response

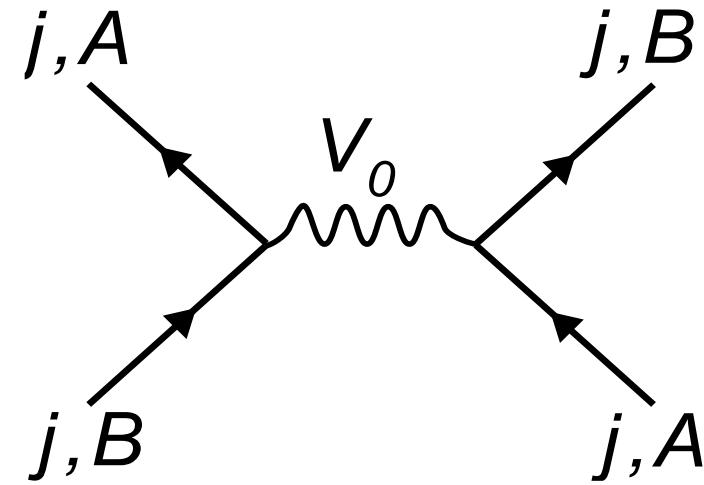
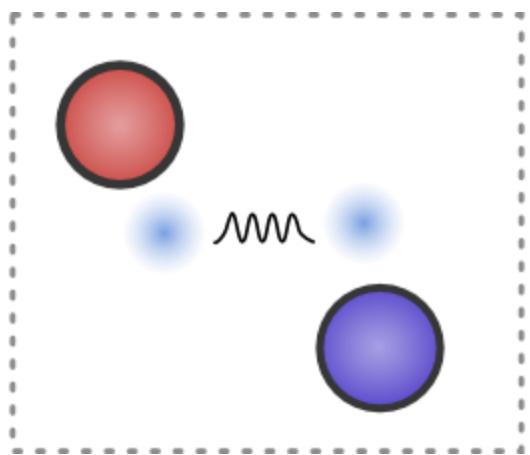
$$P = \frac{1}{L} \frac{dF}{dE}$$



$$P = \int_{-\pi}^{\pi} \frac{dk}{4\pi i} \text{tr} [\sigma_z G_0(k, 0) \partial_k G_0^{-1}(k, 0)]$$

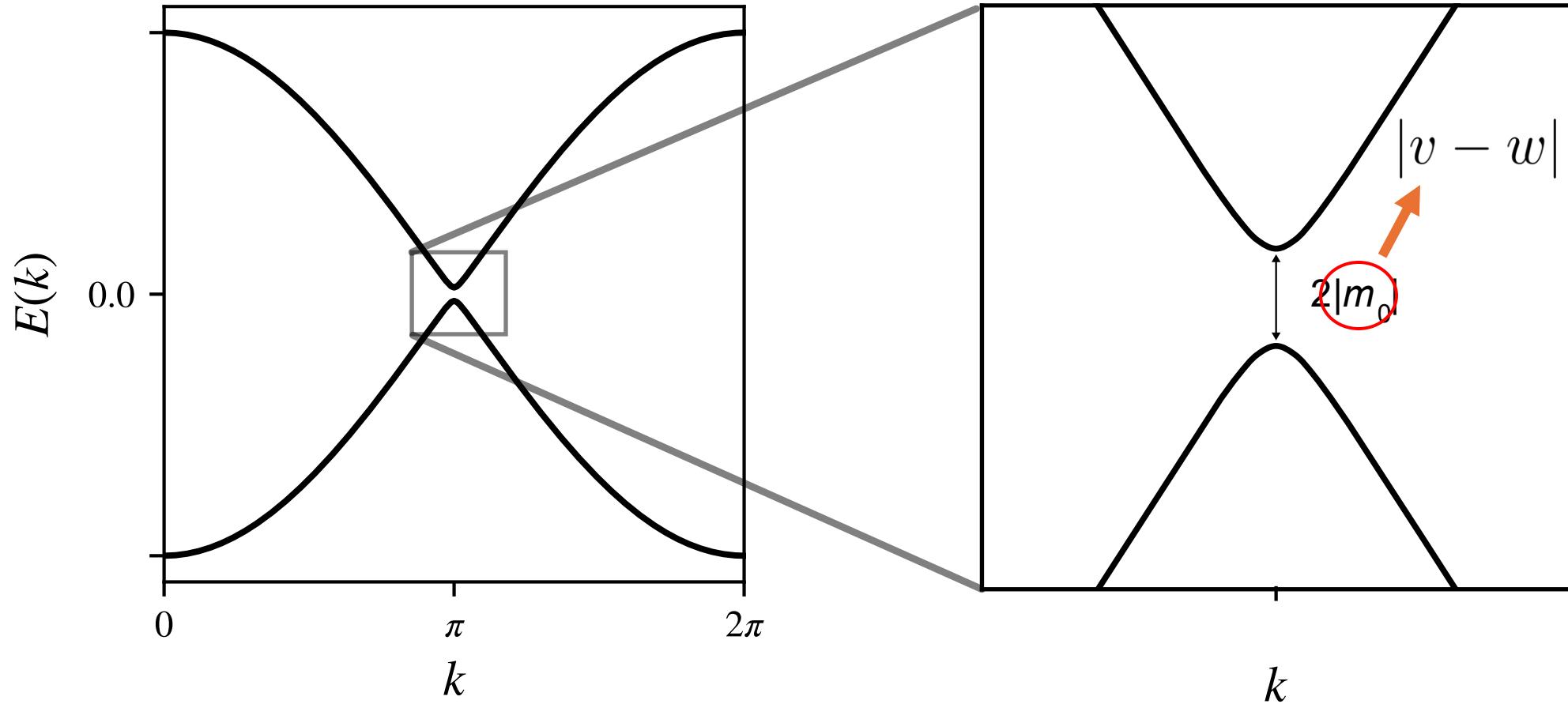
Green's function

Interactions - intracell



$$\hat{H}_{\text{int}} = V_0 \sum_j c_{j,A}^\dagger c_{j,A} c_{j,B}^\dagger c_{j,B}$$

Continuum limit – near $ka = \pi$



Continuum limit – Dirac Hamiltonian

$$\hat{H}_0 \longrightarrow \mathcal{H}_0 = \overline{\Psi}(x) (iv_F \gamma_1 \partial_x + m_0) \Psi(x)$$
$$\hat{H}_{\text{int}} \longrightarrow \mathcal{H}_I = \frac{g}{2} [\overline{\Psi}(x) \gamma_\mu \Psi(x)]^2$$
$$V_0 a/2$$
$$\gamma_0 = \sigma_x, \gamma_1 = i\sigma_z$$

wa

The diagram illustrates the transition from a discrete Hamiltonian to a continuous Dirac Hamiltonian. An orange arrow points from \hat{H}_0 to the equation $\mathcal{H}_0 = \overline{\Psi}(x) (iv_F \gamma_1 \partial_x + m_0) \Psi(x)$. Another orange arrow points from \hat{H}_{int} to the equation $\mathcal{H}_I = \frac{g}{2} [\overline{\Psi}(x) \gamma_\mu \Psi(x)]^2$. Red circles highlight the terms iv_F , g , and γ_μ . An orange arrow labeled "wa" points to the term $V_0 a/2$.

Continuum limit – massive Thirring model

$$\mathcal{L} = \overline{\Psi}(x) \left(i v_F \not{\partial} - m_0 \right) \Psi(x) - \frac{g}{2} \left[\overline{\Psi}(x) \gamma_\mu \Psi(x) \right]^2$$

Polarization – Continuum Limit $g = 0$

$$P = \int_{-\infty}^{\infty} \frac{dk}{4\pi i} \operatorname{tr} [\sigma_z G_0(k, 0) \partial_k G_0^{-1}(k, 0)]$$

$$P = \frac{1}{2} \operatorname{sign} (m_0)$$

Polarization – Continuum Limit $g \neq 0$

$$P = \int_{-\infty}^{\infty} \frac{dk}{4\pi i} \operatorname{tr} [\sigma_z G(k, 0) \partial_k G^{-1}(k, 0)]$$

$$P = \lim_{\Lambda \rightarrow \infty} [\varphi(\Lambda) - \varphi(-\Lambda)]$$



Phase of G.F.

Green's function – scaling behavior

$$G \sim |x|^{-2\Delta}$$

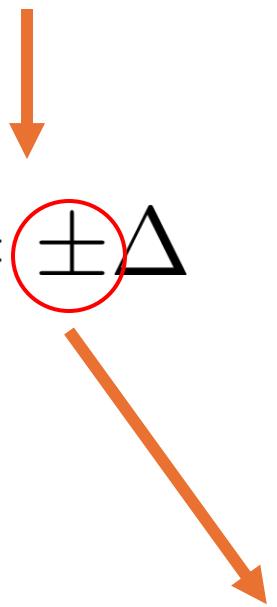
Scaling dimension of
the field in UV CFT

$$P = \pm\Delta$$

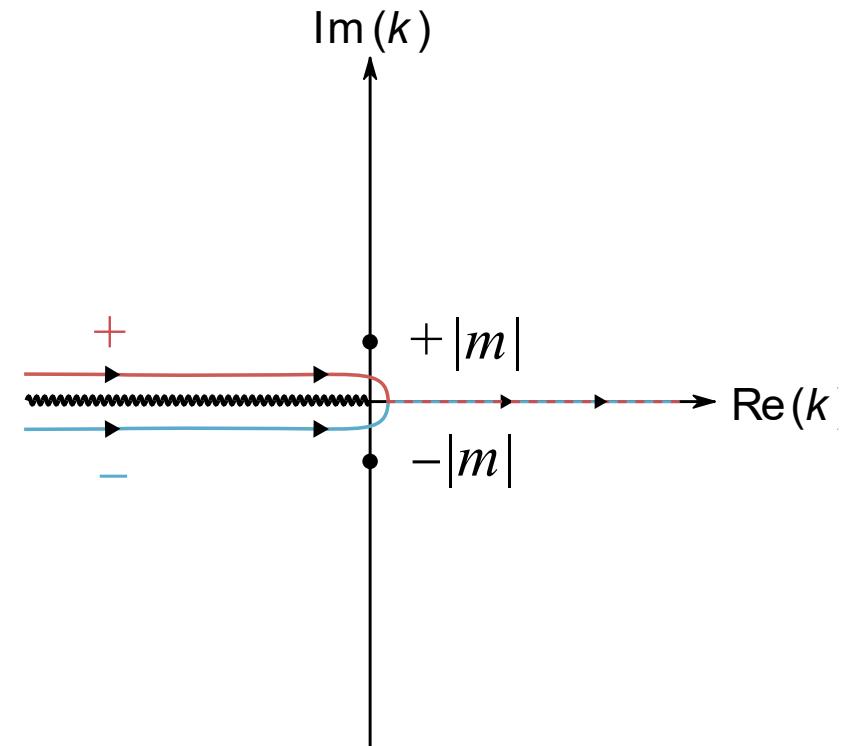
Green's function – scaling behavior

$$G \sim |x|^{-2\Delta}$$

$$P = \pm \Delta$$



Depends on path of integration!



Green's function – scaling behavior

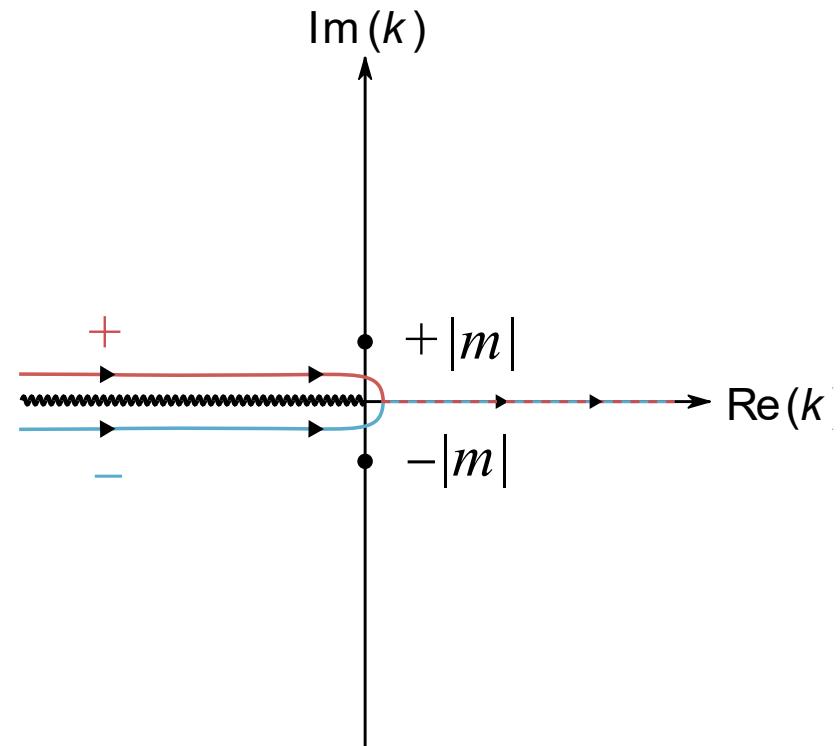
$$G \sim |x|^{-2\Delta}$$



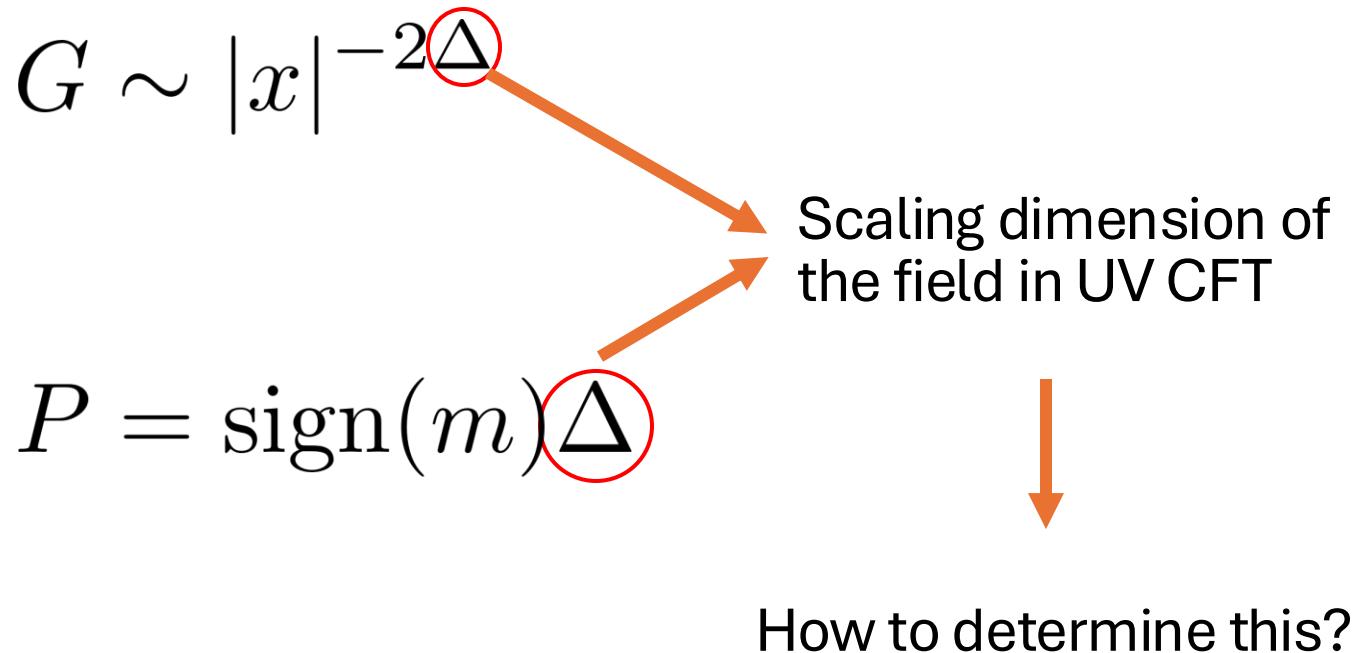
$$P = \text{sign}(m) \Delta$$



Protection provided by
IR mass scale



Green's function – scaling behavior



Duality transformation

$$\Psi(x, t) = \exp\left\{ \frac{i}{2} \left[\beta\Phi(x, t) + \frac{1}{\beta}\Theta(x, t) \right] \right\}$$

$$\overline{\Psi}(x, t) = \exp\left\{ -\frac{i}{2} \left[\beta\Phi(x, t) - \frac{1}{\beta}\Theta(x, t) \right] \right\}$$

Bosonic Fields



Sine-Gordon model

$$\mathcal{L}_{\text{SG}} = \frac{1}{2} (\partial_\mu \Phi)^2 - \frac{m_0^2}{\beta^2} [\cos(\beta\Phi) - 1]$$

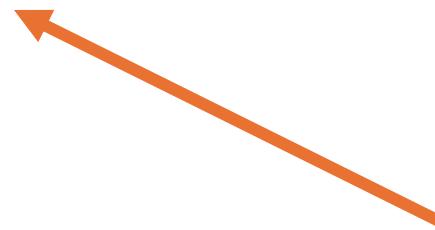
$$\frac{1}{2} \frac{1}{1 - g/\pi}$$

Green's function – form factors

$$G(k) = \sum_{n=0}^{\infty} \int_{-\infty}^{+\infty} \frac{dx}{2\pi} \frac{d\theta_1}{2\pi} \dots \frac{d\theta_n}{2\pi} e^{ikx} |F_n^{\tilde{\mathcal{O}}}(\theta_1, \dots, \theta_n)|^2$$

$F_n^{\tilde{\mathcal{O}}}(\theta_1, \dots, \theta_n)$

$$\times \delta \left(k - m \sum_{j=1}^n \sinh \theta_j \right).$$



Exact form factors in UV region yield

$$\Delta = \frac{1}{4} \left(\frac{1}{1 - g/\pi} + 1 - \frac{g}{\pi} \right)$$

Polarization of interacting model

$$P = \frac{1}{4} \left(\frac{1}{1 - g/\pi} + 1 - \frac{g}{\pi} \right)$$

String operator contribution

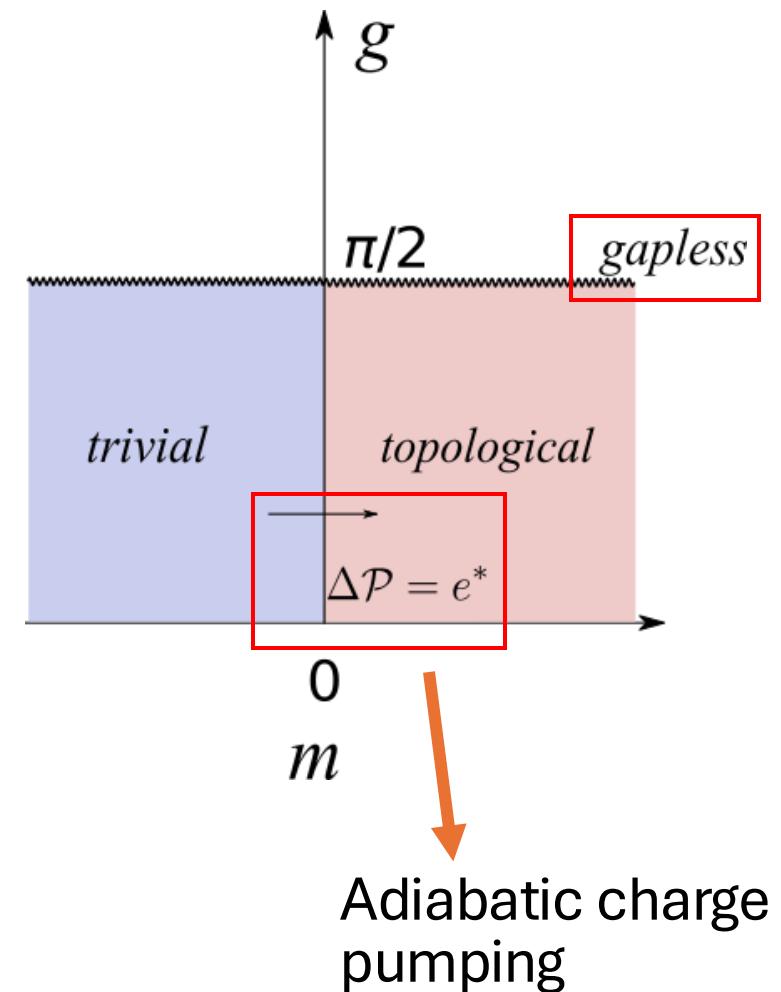
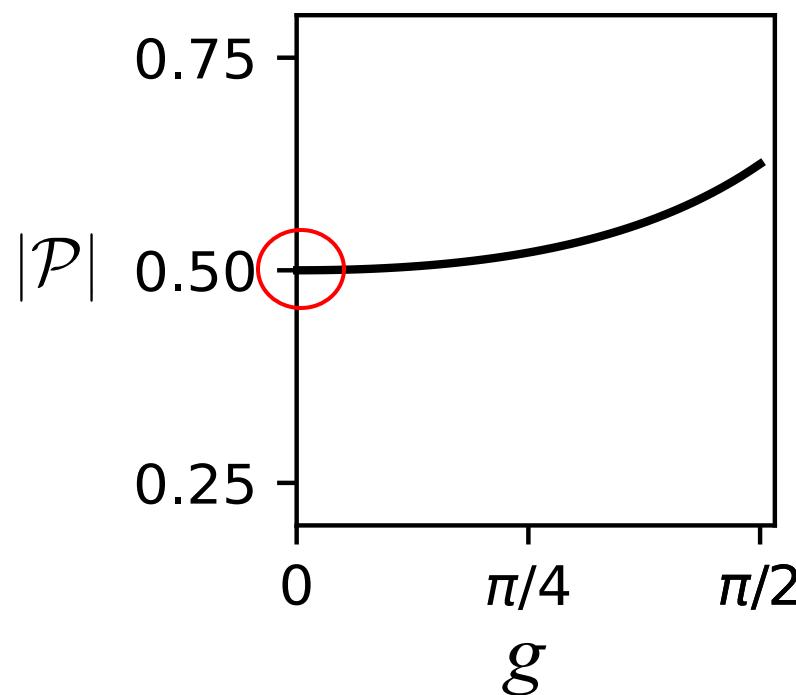
Renormalization of fermionic charge

sign(m_0)

Protection provided by IR mass scale

The diagram illustrates the decomposition of the polarization P into three components. The expression for P is shown as a sum of two terms. The first term, $\frac{1}{1 - g/\pi}$, is enclosed in a red box and has an orange arrow pointing to the text "String operator contribution". The second term, $1 - \frac{g}{\pi}$, is also enclosed in a red box and has an orange arrow pointing to the text "Protection provided by IR mass scale". Below the first term, another orange arrow points to the text "Renormalization of fermionic charge". To the right of the second term, there is a red circle containing the text "sign(m_0)", with an orange arrow pointing downwards from it.

Polarization of interacting model



Further Insights – RG

$$\left[\Lambda \frac{\partial}{\partial \Lambda} + \sum_i \beta_{\text{RG}}(g_i) \frac{\partial}{\partial g_i} + n\gamma \right] G^{(n)}(k_1, \dots, k_n; \Lambda, g) = 0$$



Scaling dimension of field

$$\beta_{RG} \rightarrow 0 \quad \longrightarrow \quad \text{Im log } G^{(2)} \sim 2\gamma \text{Im log}(k/\Lambda)$$

Take-home message

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- Study interacting SSH using polarization
- Continuum limit to obtain Thirring model
- Exact results provide insights on topological phase
- Topological phase protected by IR (mass)
- Magnitude of Polarization determined by UV (scaling dimension)

Team



Labor force



Supervisors

