Topological Phases of the interacting SSH model

Presenter: Anouar Moustaj

Supervisor: Cristiane Morais Smith

In collaboration with

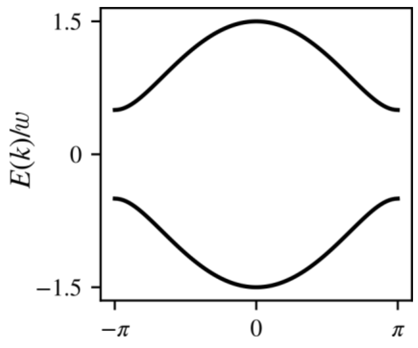
Emanuele di Salvo, **Chen Xu**,

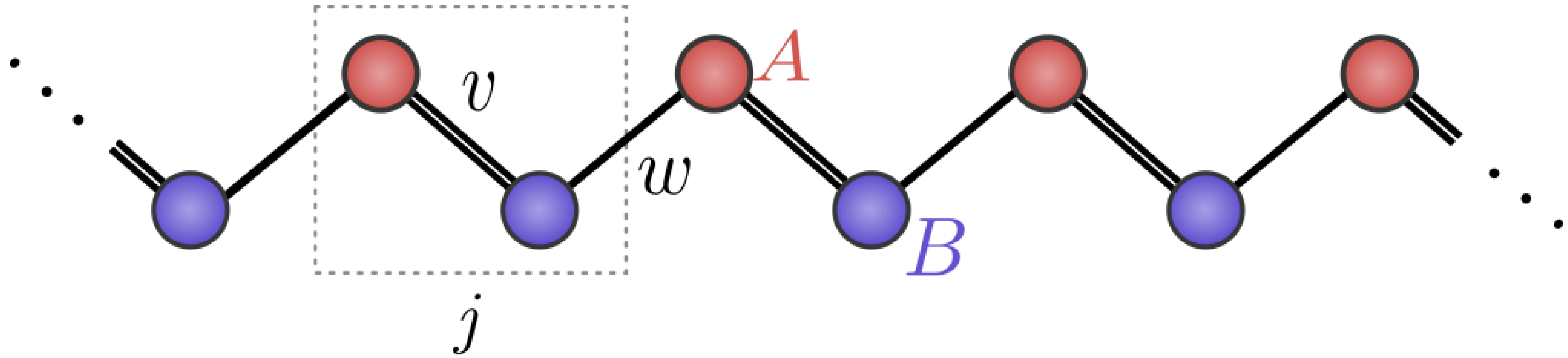
Lars Fritz, Andrew Mitchell, Dirk Schuricht

Phys. Rev. B **110**, 165145

arXiv: 2408.01421

SSH Model





$$\hat{H}_0 = v \sum_j c_{j,A}^\dagger c_{j,B} + w \sum_j c_{j,B}^\dagger c_{j+1,A} + \text{h.c.}$$

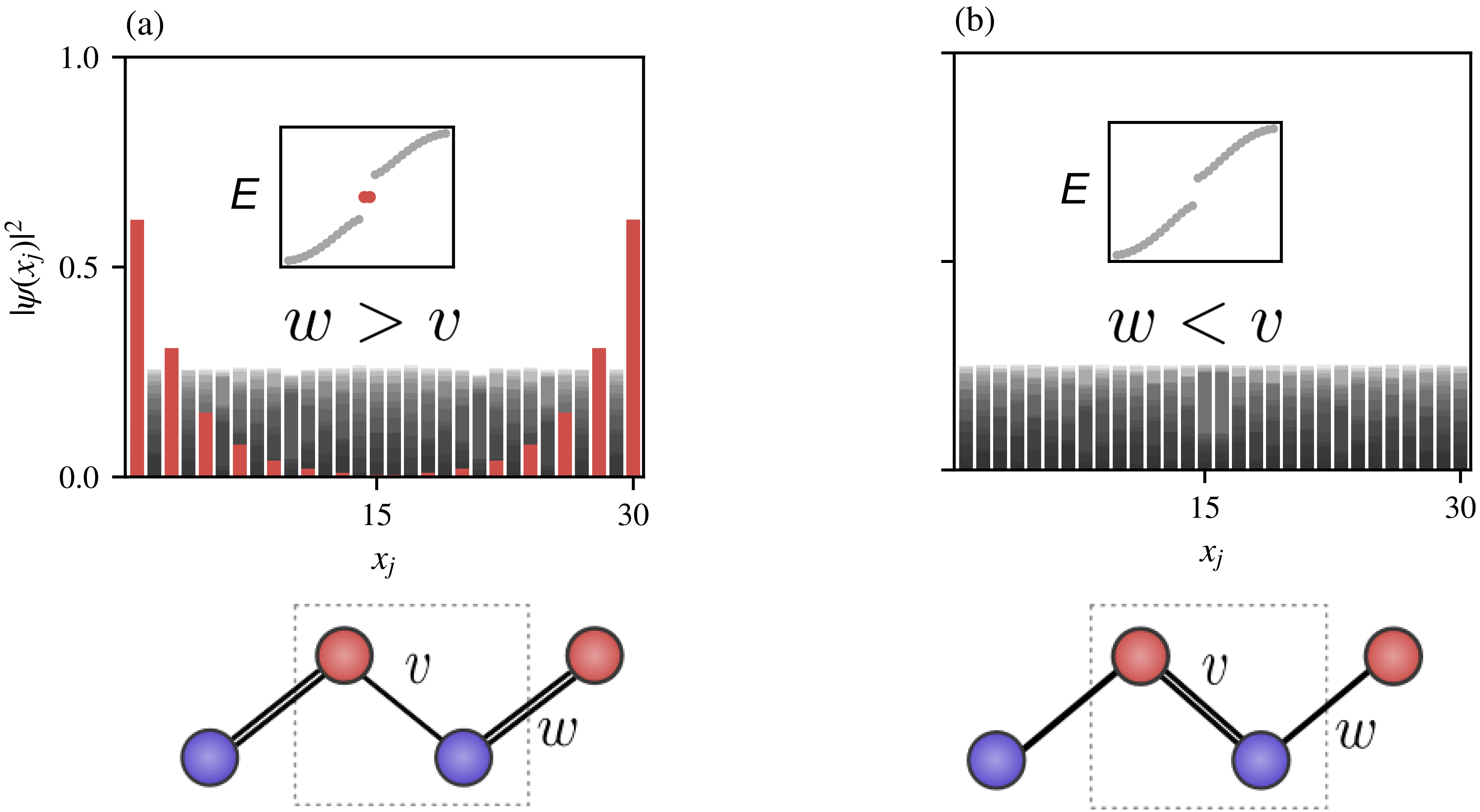
Topology - Chiral symmetry

$$[H(k), \sigma_z]_+ = 0$$

$$\nu = \int_{-\pi}^{\pi} \frac{dk}{4\pi i} \mathrm{Tr}\left[\sigma_z H^{-1}(k) \frac{dH(k)}{dk}\right] = \begin{cases} 1, & \text{if } w > v \\ 0, & \text{if } w < v \end{cases}$$

Rev. Mod. Phys. **88**, 035005 (2016)

Bulk-Boundary Correspondence



Physical Observable - Polarization

$$P = e\langle \hat{X} \rangle$$

Modern Theory

Berry phase →

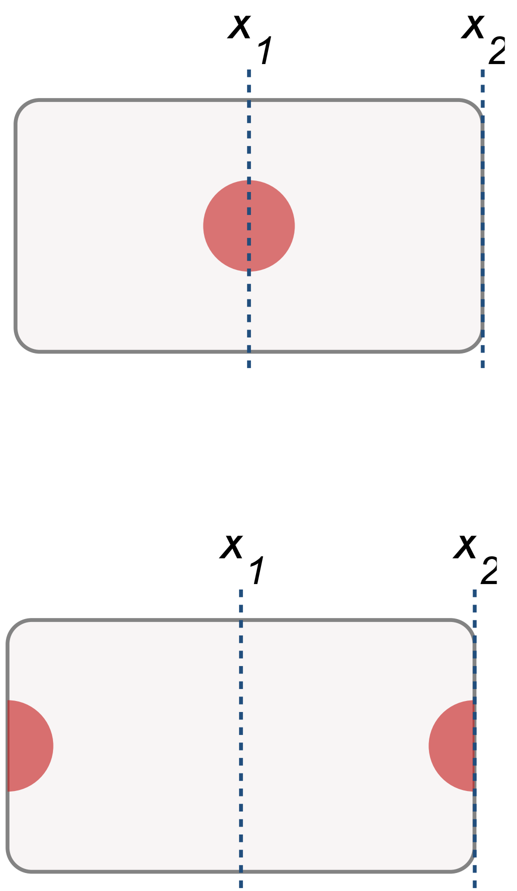
$$P = e\frac{\phi}{2\pi} \mod e$$

Phys. Rev. B **47**, 1651(R) (1993)

Polarization quantization

Inversion symmetry

$\phi = 0, \pi$ — $P = 0, \frac{1}{2}$



$P = 0$

$P = \frac{1}{2}$

Phys. Rev. Lett. **62**, 2747 (1989)

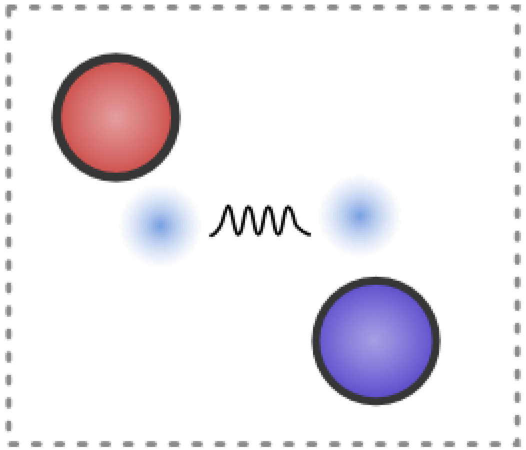
Polarization – topological response

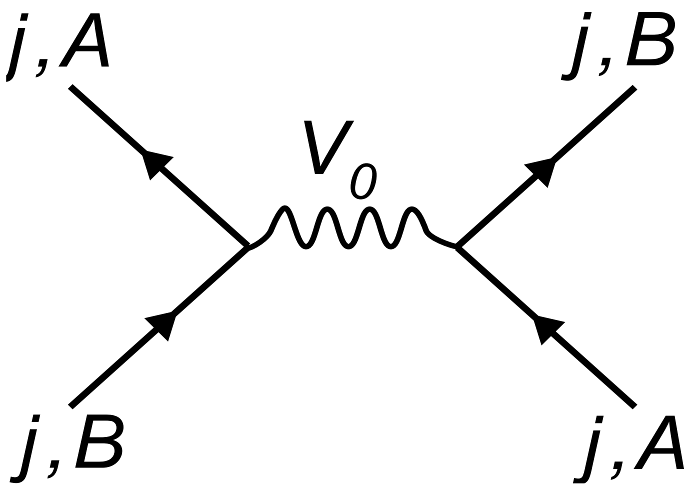
$$P = \frac{1}{L}\frac{\mathrm{d}F}{\mathrm{d}E}$$

$$P = \int_{-\pi}^{\pi} \frac{\mathrm{d}k}{4\pi i} \,\mathrm{tr}\left[\sigma_z G_0(k,0)\partial_k G_0^{-1}(k,0)\right]$$

Green's function

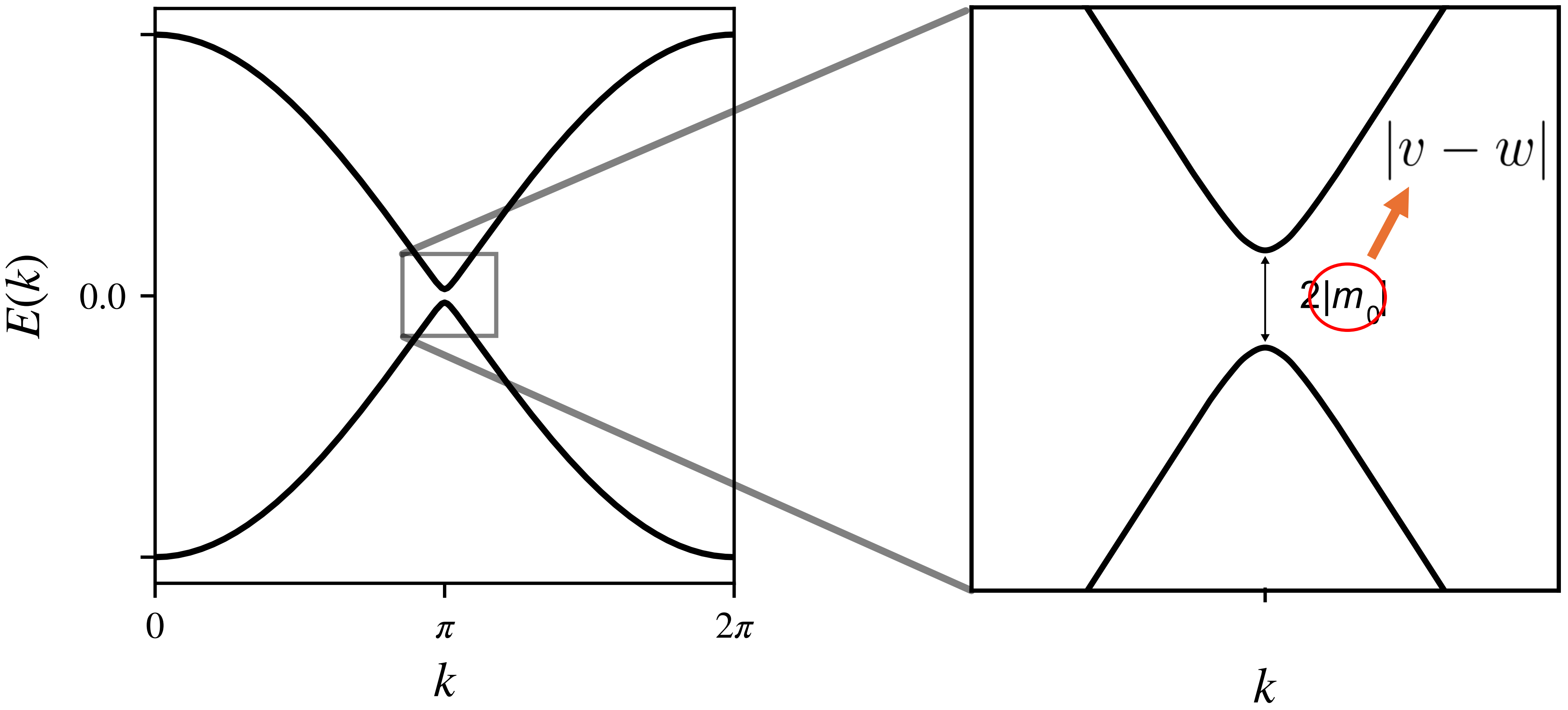
Interactions - intracell





$$\hat{H}_{\mathrm{int}} = V_0 \sum_j c^\dagger_{j,A} c_{j,A} c^\dagger_{j,B} c_{j,B}$$

Continuum limit – near $ka = \pi$



Continuum limit – Dirac Hamiltonian

wa

$\hat{H}_0 \longrightarrow \mathcal{H}_0 = \overline{\Psi}(x)\left(i v_F \gamma_1 \partial_x + m_0\right)\Psi(x)$

$\hat{H}_{\text{int}} \longrightarrow \mathcal{H}_I = \frac{g}{2}\left[\overline{\Psi}(x)\gamma_\mu \Psi(x)\right]^2$

$V_0 a/2$

$\gamma_0 = \sigma_x, \gamma_1 = i\sigma_z$

Continuum limit – massive Thirring model

$$\mathcal{L} = \overline{\Psi}(x)\left(iv_F \not{\partial} - m_0\right)\Psi(x) - \frac{g}{2}\left[\overline{\Psi}(x)\gamma_\mu \Psi(x)\right]^2$$

Ann. Phys. **3**, 91 (1958), Theor. Math. Phys. **41**, 953 (1979)

Polarization – Continuum Limit $\quad g = 0$

$$P = \int_{-\infty}^{\infty} \frac{\mathrm{d}k}{4\pi i} \,\mathrm{tr}\left[\sigma_z G_0(k,0) \partial_k G_0^{-1}(k,0)\right]$$

$$P = \frac{1}{2}\mathrm{sign}\left(m_0\right)$$

Polarization – Continuum Limit $g \neq 0$

$$P = \int_{-\infty}^{\infty} \frac{\mathrm{d}k}{4\pi i} \operatorname{tr}\left[\sigma_z G(k,0)\partial_k G^{-1}(k,0)\right]$$

$$P = \lim_{\Lambda\to\infty}\left[\varphi(\Lambda) - \varphi(-\Lambda)\right]$$

Phase of G.F.

Green's function – scaling behavior

$$G \sim |x|^{-2\Delta}$$

Scaling dimension of the field in UV CFT

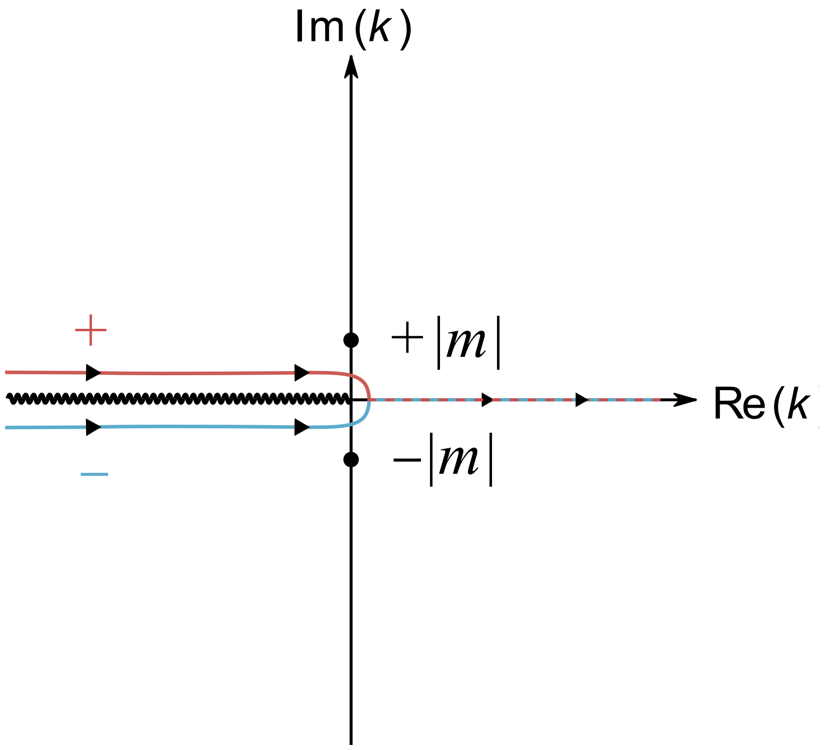
$$P = \pm\Delta$$

Green's function – scaling behavior

$$G \sim |x|^{-2\Delta}$$

$$P = \pm\Delta$$

Depends on path of integration!

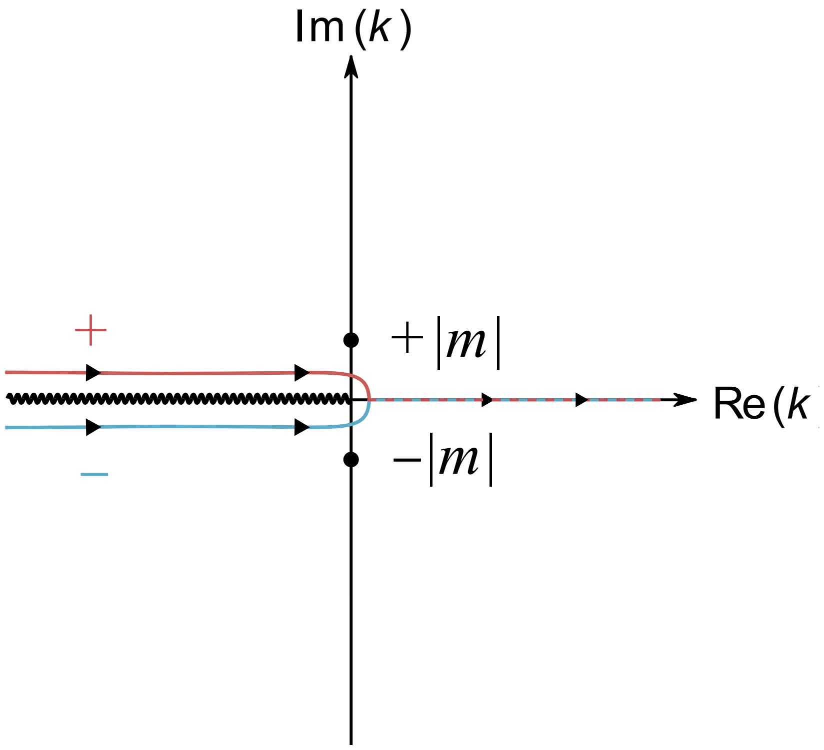


Green’s function – scaling behavior

$$G \sim |x|^{-2\Delta}$$

$$P = \mathrm{sign}(m)\Delta$$

Protection provided by
IR mass scale



Green’s function – scaling behavior

$$G \sim |x|^{-2\Delta}$$

$$P = \text{sign}(m)\Delta$$

Scaling dimension of the field in UV CFT

How to determine this?

Duality transformation

$$\Psi(x,t) = \exp\left\{\frac{i}{2}\left[\beta\Phi(x,t) + \frac{1}{\beta}\Theta(x,t)\right]\right\}$$

$$\overline{\Psi}(x,t) = \exp\left\{-\frac{i}{2}\left[\beta\Phi(x,t) - \frac{1}{\beta}\Theta(x,t)\right]\right\}$$

Bosonic Fields

Sine-Gordon model

$$\mathcal{L}_{\mathrm{SG}} = \frac{1}{2}\left(\partial_\mu \Phi\right)^2 - \frac{m_0^2}{\beta^2}\left[\cos\left(\beta\Phi\right) - 1\right]$$

$$\frac{1}{2}\frac{1}{1-g/\pi}$$

Green's function – form factors

$$G(k) = \sum_{n=0}^{\infty} \int_{-\infty}^{+\infty} \frac{\mathrm{d}x}{2\pi} \frac{\mathrm{d}\theta_1}{2\pi} \cdots \frac{\mathrm{d}\theta_n}{2\pi} e^{ikx} |F_n^{\tilde{\mathcal{O}}}(\theta_1, \ldots, \theta_n)|^2$$

$$\times \, \delta\left(k - m \sum_{j=1}^{n} \sinh \theta_j\right) .$$

Exact form factors in UV region yield

$$\Delta = \frac{1}{4}\left(\frac{1}{1-g/\pi}+1-\frac{g}{\pi}\right)$$

Nucl. Phys. B **607**, 437 (2001)

Polarization of interacting model

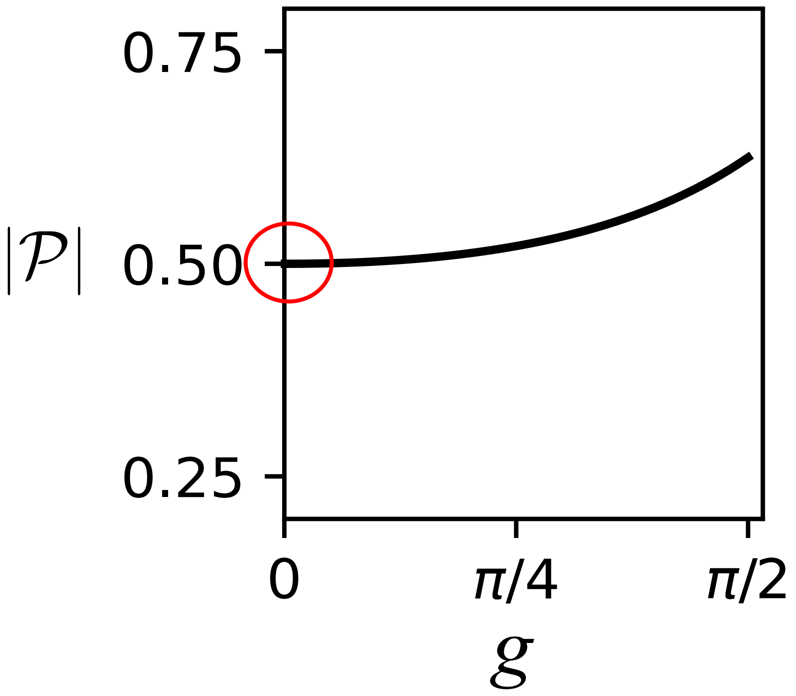
$$P = \frac{1}{4}\left(\frac{1}{1-g/\pi} + 1 - \frac{g}{\pi}\right)\operatorname{sign}(m_0)$$

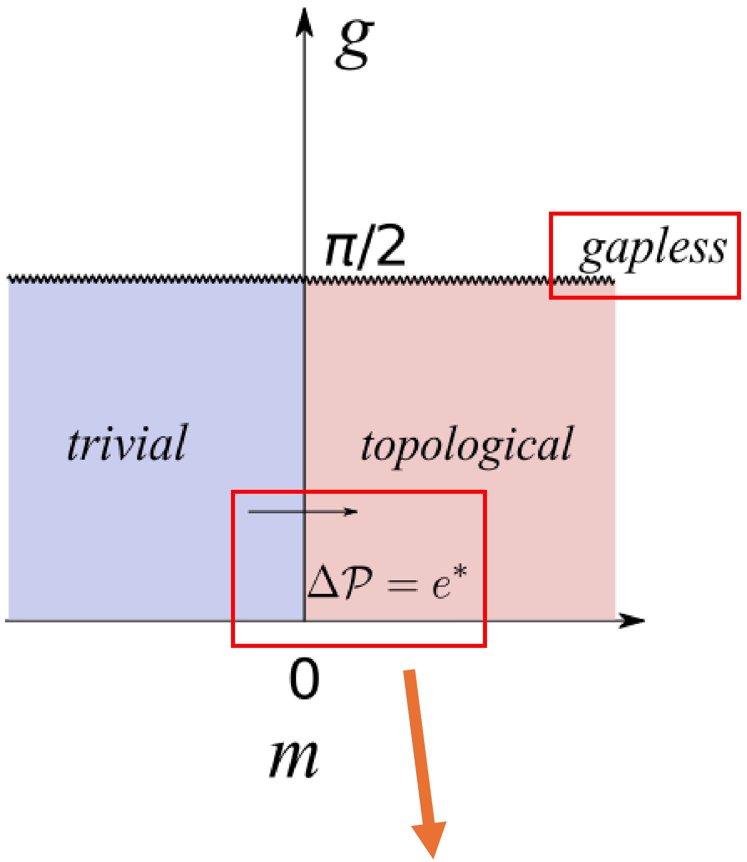
String operator contribution

Renormalization of fermionic charge

Protection provided by IR mass scale

Polarization of interacting model





Adiabatic charge pumping

Further Insights – RG

$$\left[\Lambda\frac{\partial}{\partial\Lambda}+\sum_i\beta_{\mathrm{RG}}(g_i)\frac{\partial}{\partial g_i}+n\gamma\right]G^{(n)}(k_1,\ldots,k_n;\Lambda,g)=0$$

Scaling dimension of field

$$\beta_{RG}\to 0 \quad \longrightarrow \quad \mathrm{Im}\log G^{(2)}\sim 2\gamma\,\mathrm{Im}\log(k/\Lambda)$$

Take-home message

Phys. Rev. B **110**, 165145

arXiv: 2408.01421

- Study interacting SSH using polarization
- Continuum limit to obtain Thirring model
- Exact results provide insights on topological phase
- Topological phase protected by IR (mass)
- Magnitude of Polarization determined by UV (scaling dimension)

Team

Labor force

Supervisors