$\mathbb{Z}_4$ transitions

in quantum loop models on a zig-zag ladder

QuMat Yearly meeting

11-11-2024

QuMat

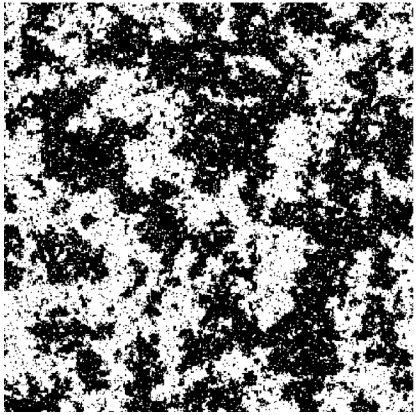
TU Delft

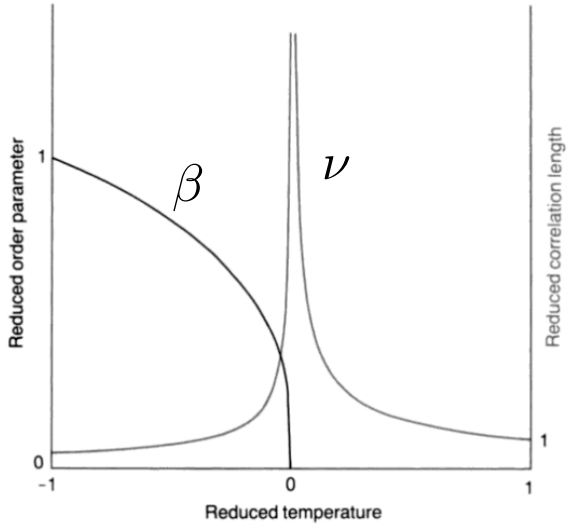
Department of Quantum Nanoscience
Opening the quantum world for innovation

Universality classes

- Dimension & symmetry
- Critical exponents
- Break $\mathbb{Z}_2 \to$ Ising
- 1D quantum magnets?
- Spin-1 chains

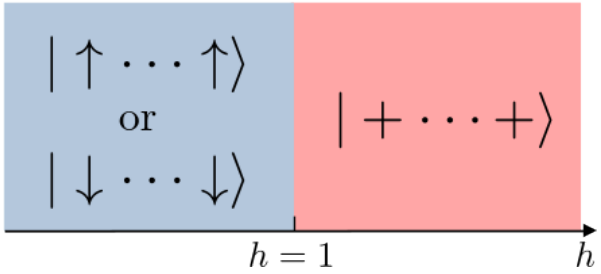
2D classical Ising



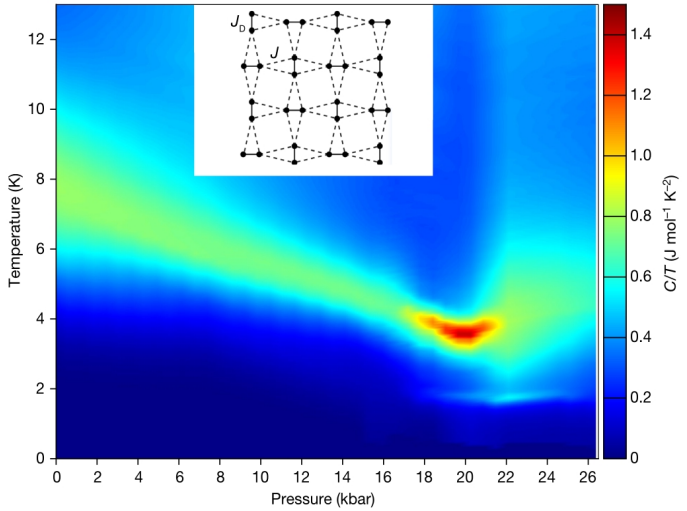


Paul Davies, "The new physics" (1989)

1D quantum Ising



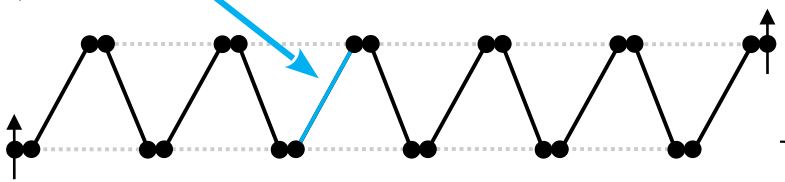
SrCu2(BO3)2



Larrea Jimenez, *et al.* Nature **592** 370-375 (2021)

Topologically non-trivial → No Ising

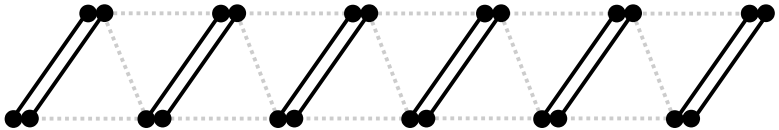
$\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) = \text{dimer}$



Disordered, Haldane (gapped)

Topologically non-trivial

Break $\mathbb{Z}_2$



$\mathbb{Z}_2$, Dimerized (gapped)

Topologically trivial

Critical behaviour of spin-*s* Heisenberg antiferromagnetic chains: analytic and numerical results

Ian Affleck†, Doron Gepner‡, H J Schulz§ and Timothy Ziman‖

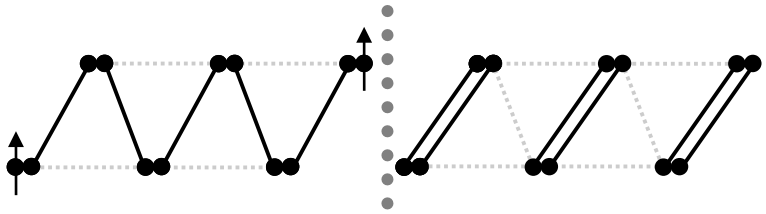
† Canadian Institute for Advanced Research and Physics Department, University of British Columbia, Vancouver, BC, Canada V6T 2A6

‡ Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544, USA

§ Laboratoire de Physique des Solides, Universite de Paris-Sud, 91405 Orsay, France

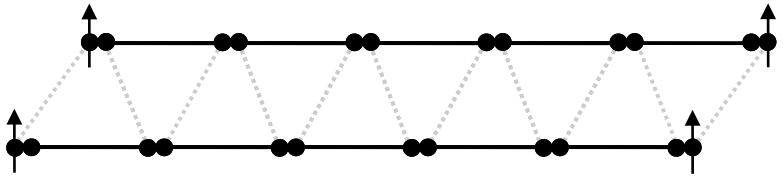
‖ Department of Physics and Astronomy, Rutgers University, Piscataway, NJ 08855, USA

Received 28 June 1988

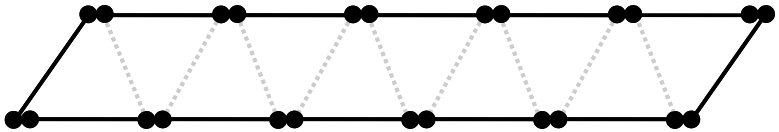


Wess-Zumino-Witten $SU(2)_2$

Topologically trivial → Ising

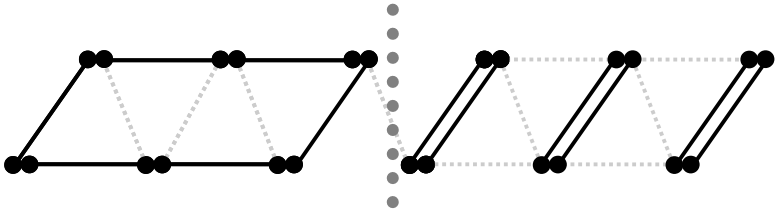


2×Haldane (gapped)
Topologically non-trivial

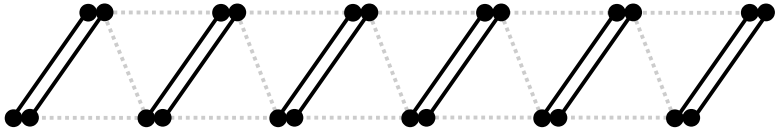


Disordered, Next-Nearest-Neighbor
(NNN) Haldane (gapped)
Topologically trivial

Break $\mathbb{Z}_2$



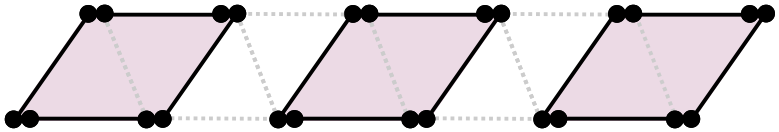
Ising!



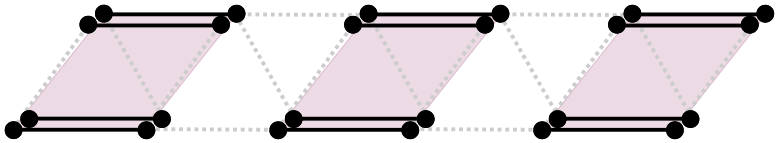
$\mathbb{Z}_2$, Dimerized (gapped)
Topologically trivial

What if we break another $\mathbb{Z}_2$ symmetry?

Plaquette (gapped)
Topologically trivial



Columnar (gapped)
Topologically trivial



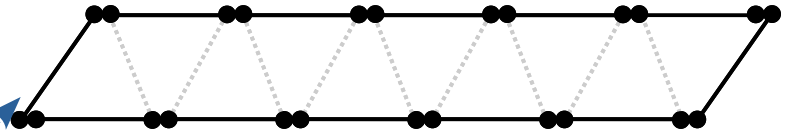
?????

Break $\mathbb{Z}_4$

$\mathbb{Z}_4$ - phase

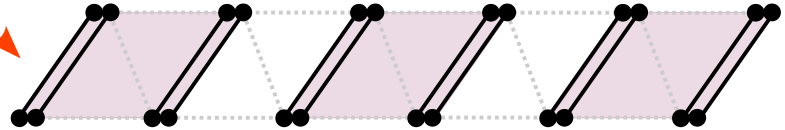
Break $\mathbb{Z}_2$

Ising?



Disordered, Next-Nearest-Neighbor
(NNN) Haldane (gapped)
Topologically trivial

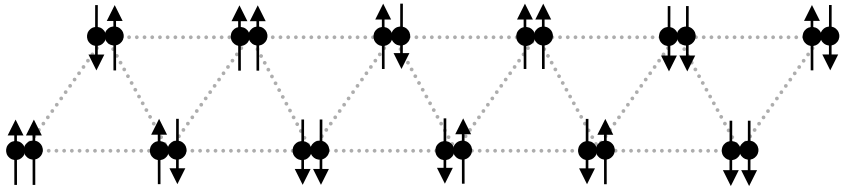
Break $\mathbb{Z}_2$



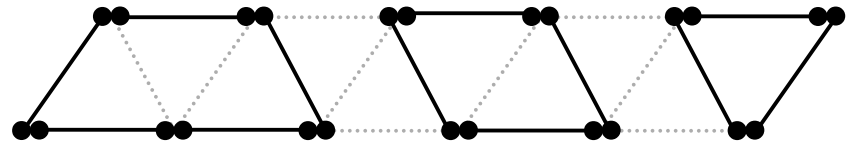
$\mathbb{Z}_2$, Dimerized (gapped)
Topologically trivial

A toy model to study quantum spin chains

quantum spin chain

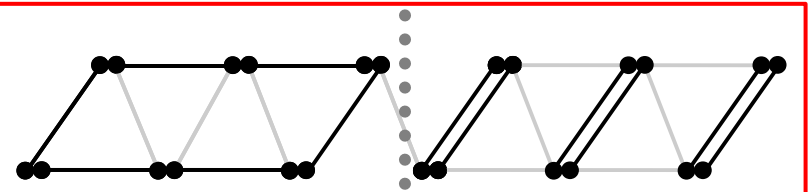


quantum loop model (QLM)

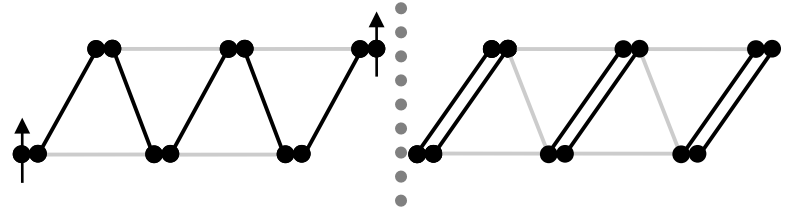


- Singlet sector
- Constrained Hilbert space:
  dim $\mathcal{H} = 3^N$ vs. dim $\mathcal{H} \approx 1.68^N$

Topologically trivial

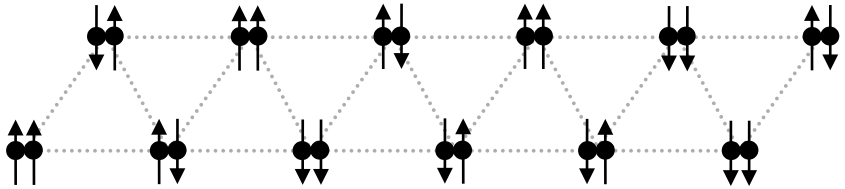


Topologically non-trivial

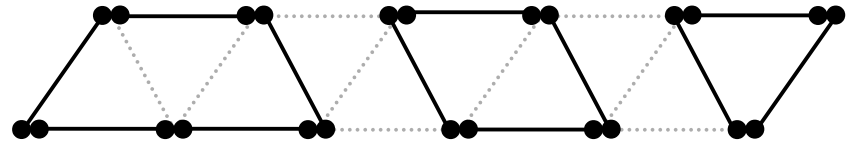


A toy model to study quantum spin chains

quantum spin chain



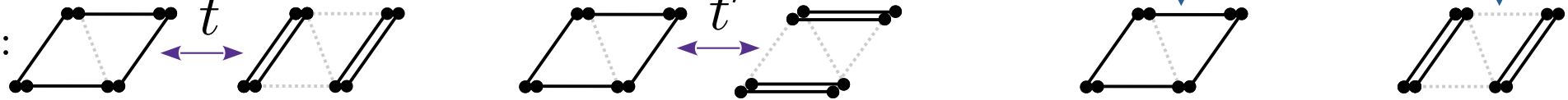
quantum loop model (QLM)



$$\mathcal{H}_{\mathrm{QLM}} = -\sum_{\text{plaquettes}} \left(t\,|\square\rangle\langle\square| + t'\,|\square\rangle\langle\square| + \text{h.c} \right) - \theta \sum_{\text{plaquettes}} |\square\rangle\langle\square| + \delta \sum_{\text{rungs}} |\,/\!/\,\rangle\langle\,/\!/\,|$$

$t' = t$

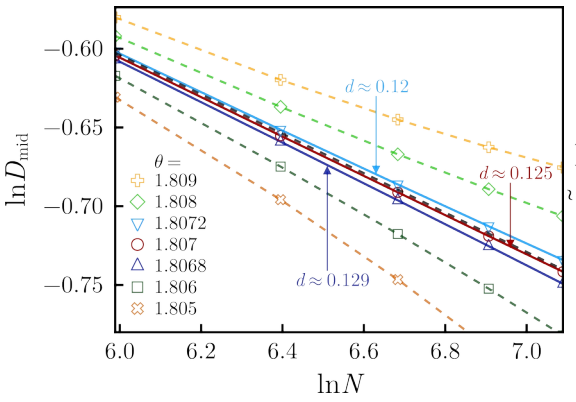
1:

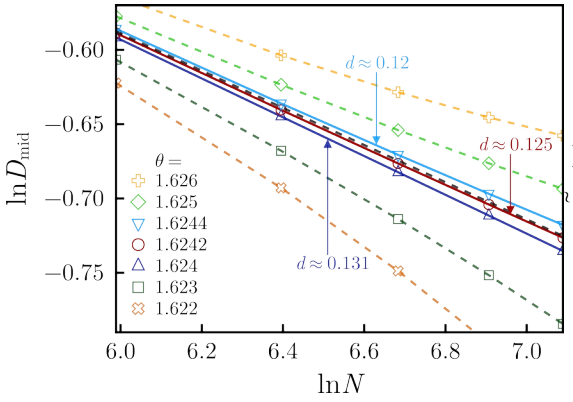


2: etc...

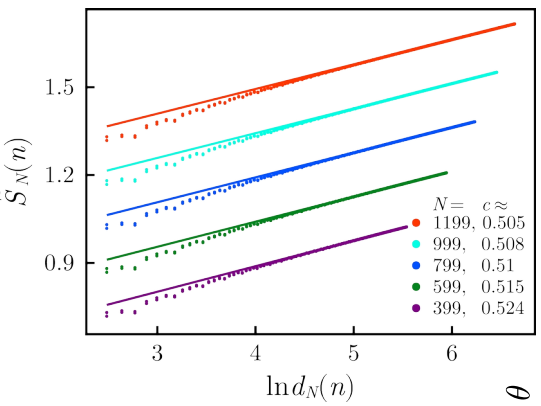
Intermediate $\mathbb{Z}_2$ phase and two Ising transitions

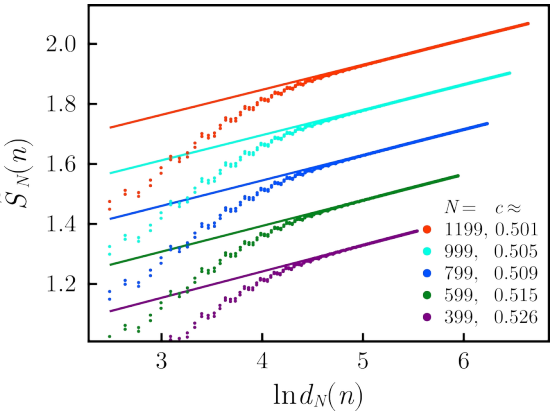
Ordered parameter

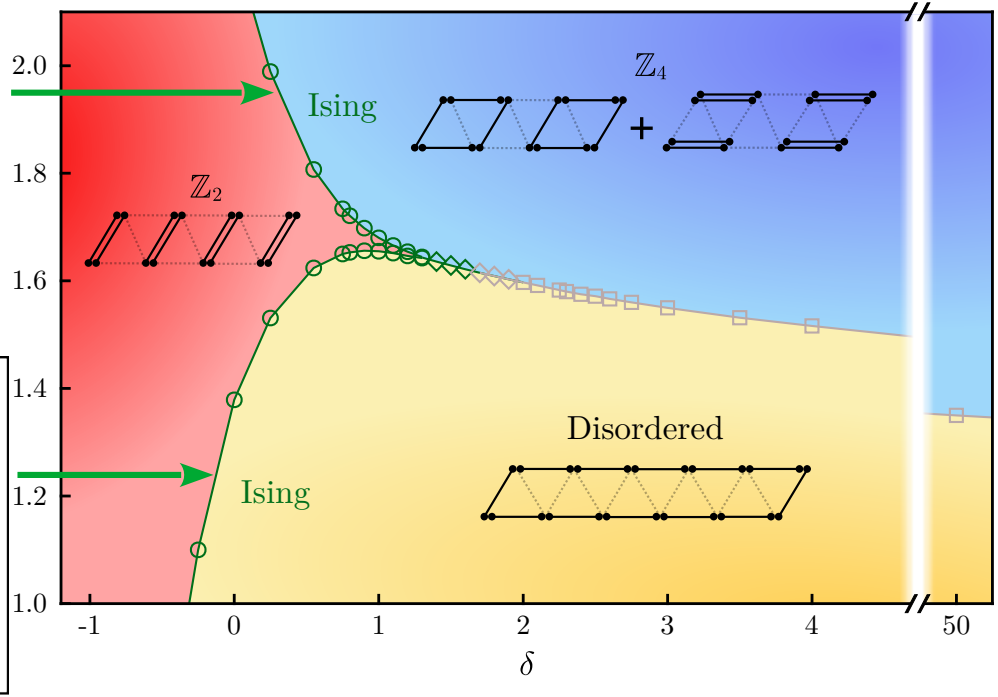




Central charge



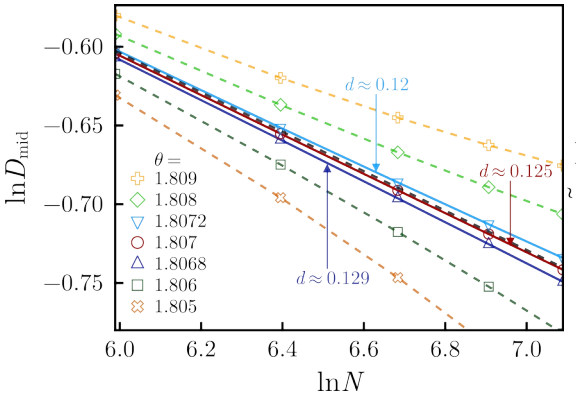


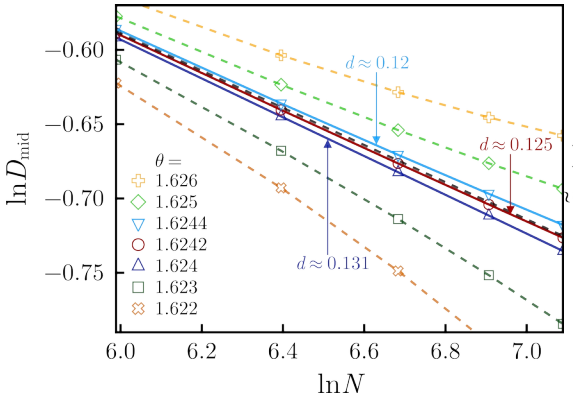


arXiv:2406.20093

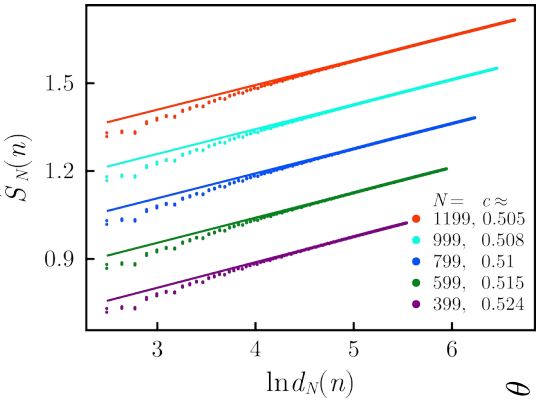
Symmetry point of view → Ashkin-Teller

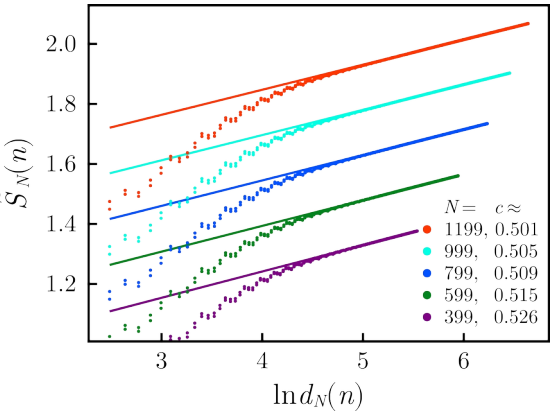
Ordered parameter

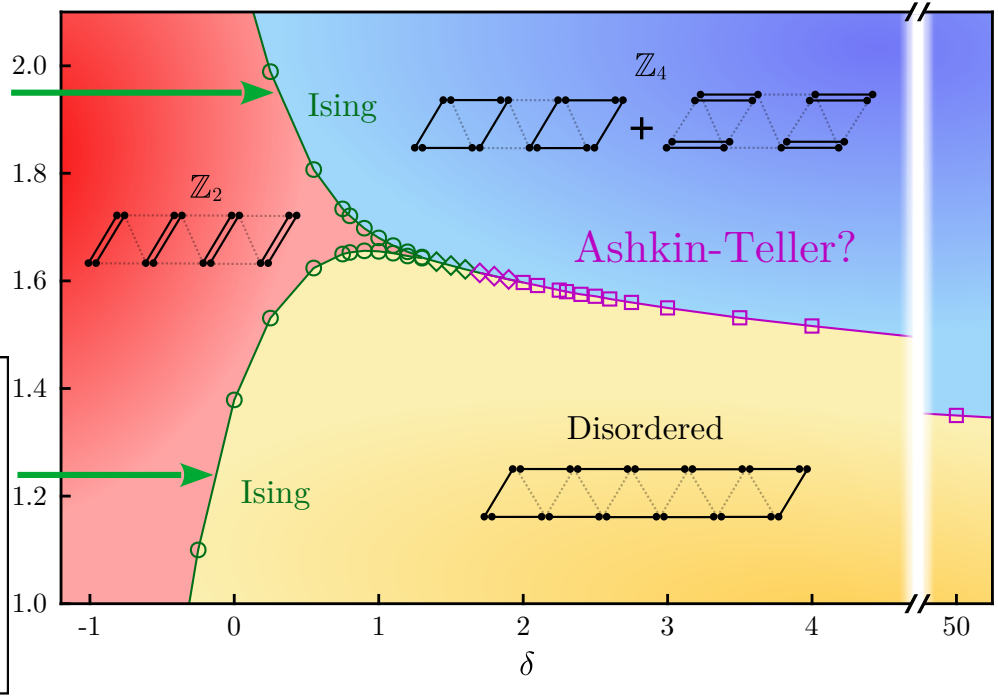




Central charge

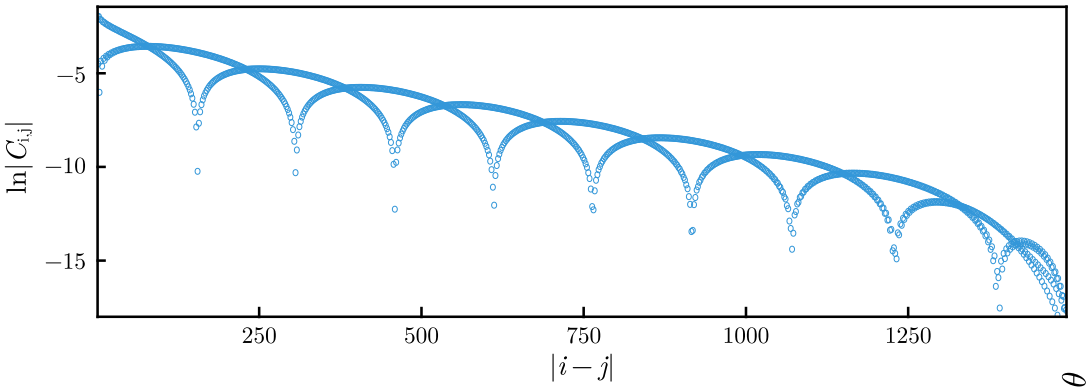




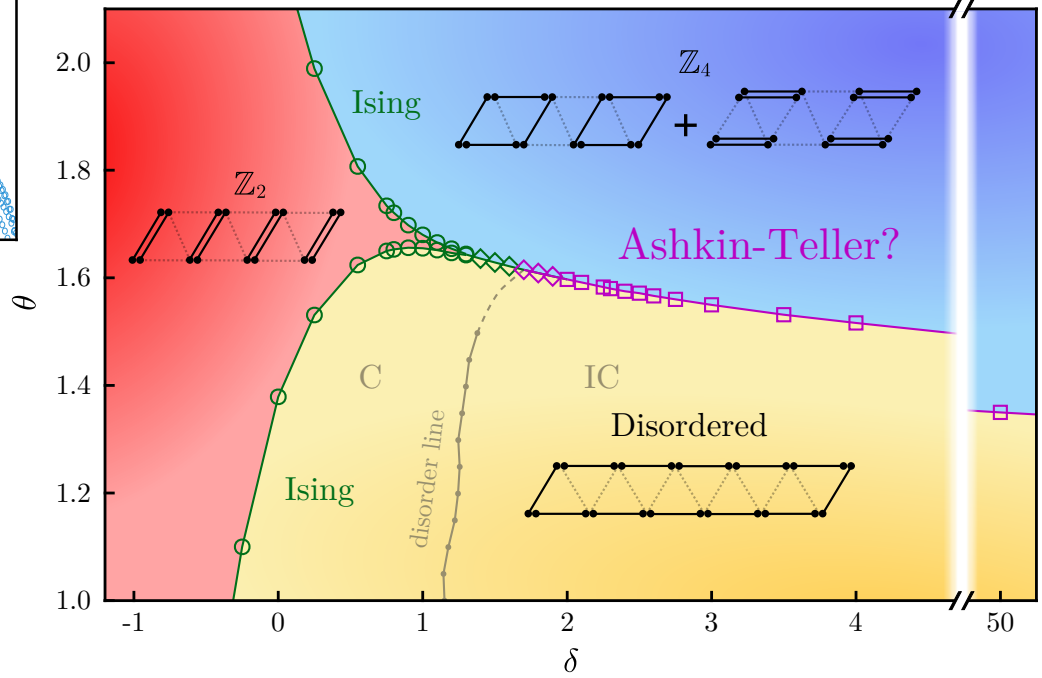


arXiv:2406.20093

What if symmetry alone is not enough?

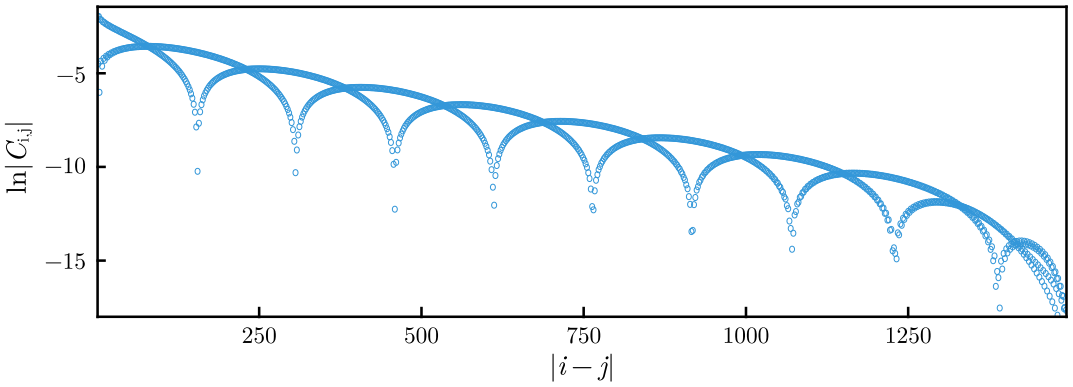


- Commensurate (C): $q = \pi/2$
- Incommensurate (IC): $q \neq \pi/2$
- Disorder line
- **Commensurate-Incommensurate transition**



arXiv:2406.20093

$\mathbb{Z}_4$ Commensurate-Incommensurate transition



$\xi \propto |\theta - \theta_c|^{-\nu}$: correlation length

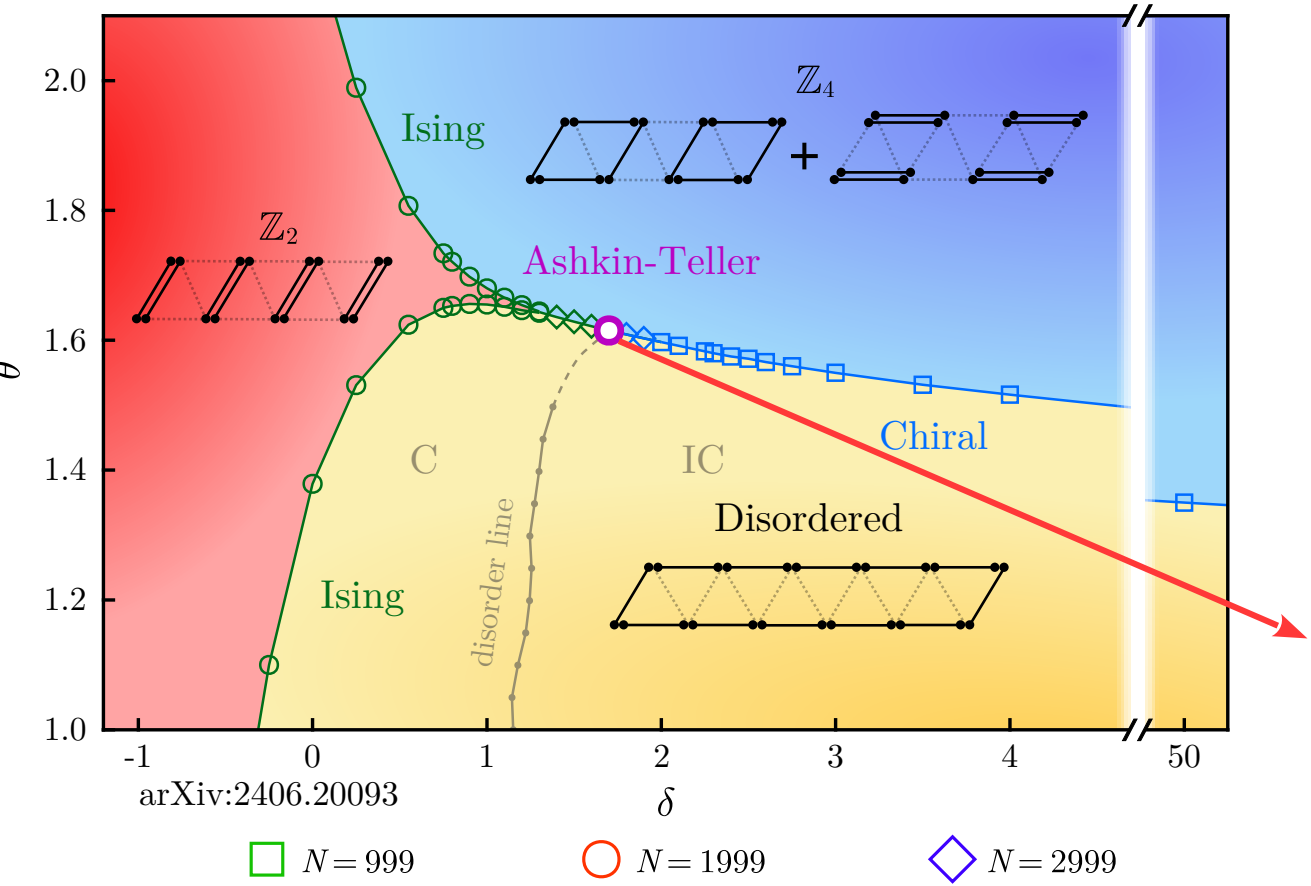
$|\Delta q| \propto |\theta - \theta_c|^{\bar{\beta}}$: distance to $q = \pi/2$

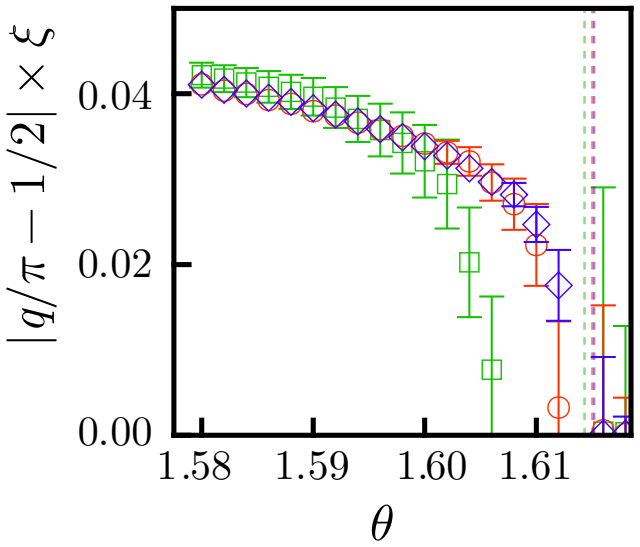
- Ashkin-Teller: $\bar{\beta} > \nu$
  $|\Delta q| \times \xi \to 0$
- Chiral: $\bar{\beta} = \nu$
  $|\Delta q| \times \xi \to$ constant
- Floating phase:
  $|\Delta q| \times \xi \to \infty$

Huse, Fisher, Phys. Rev. Lett. **49** 793 (1982)

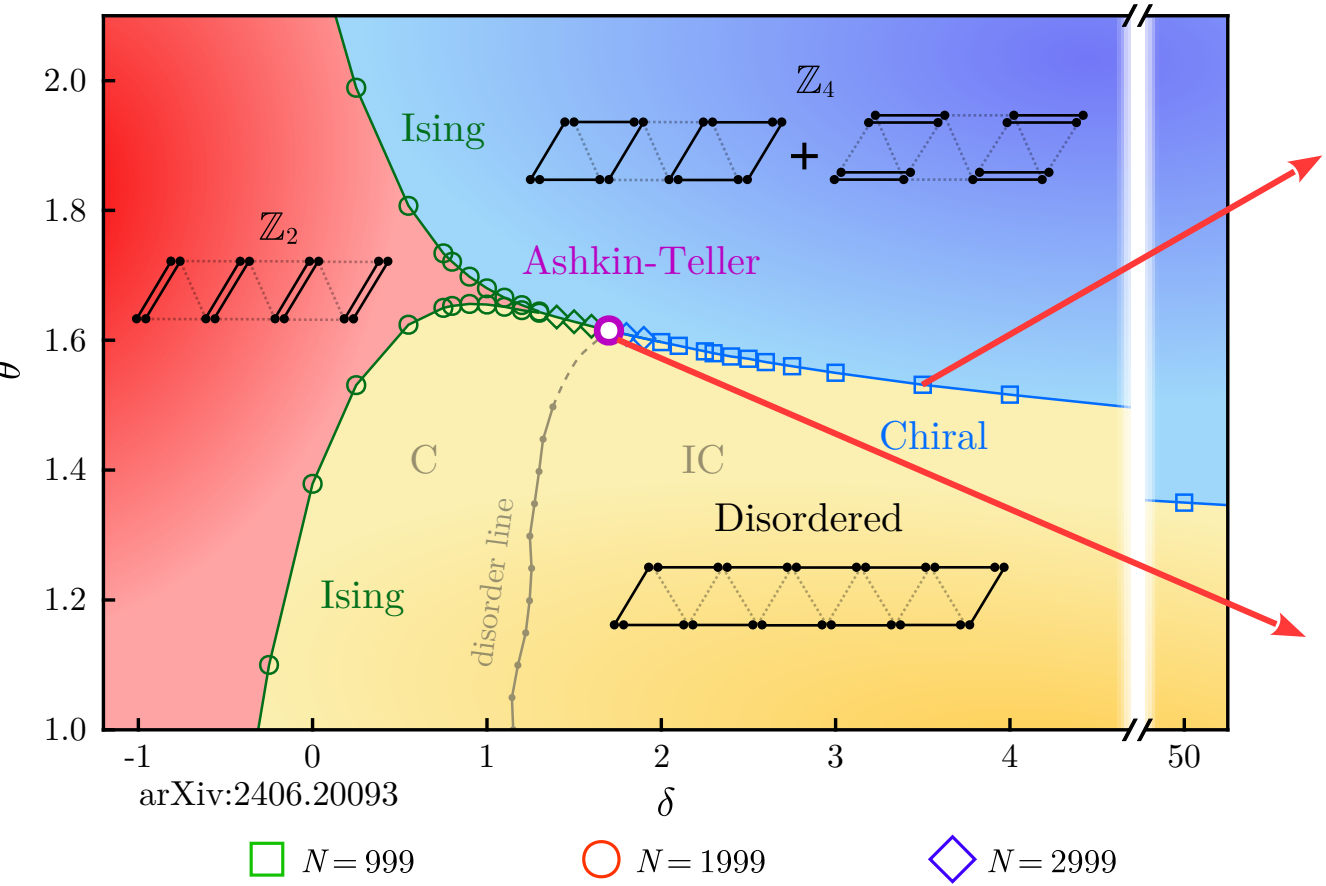
Huse, Fisher, Phys. Rev. B **29** 293 (1984)

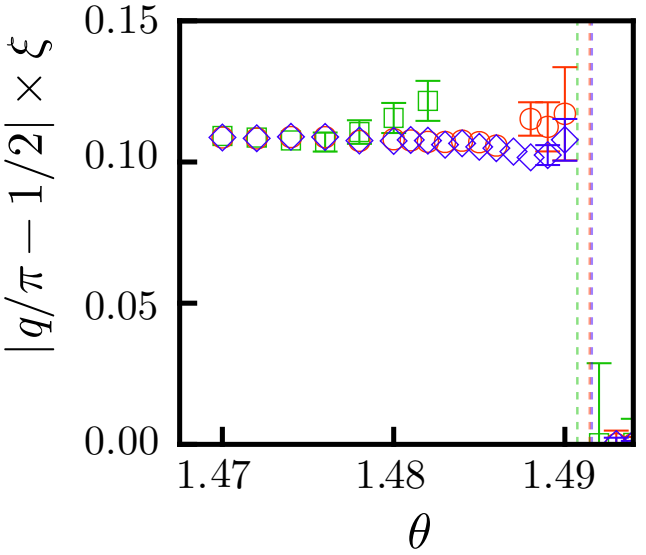
Ashkin-Teller point and chiral

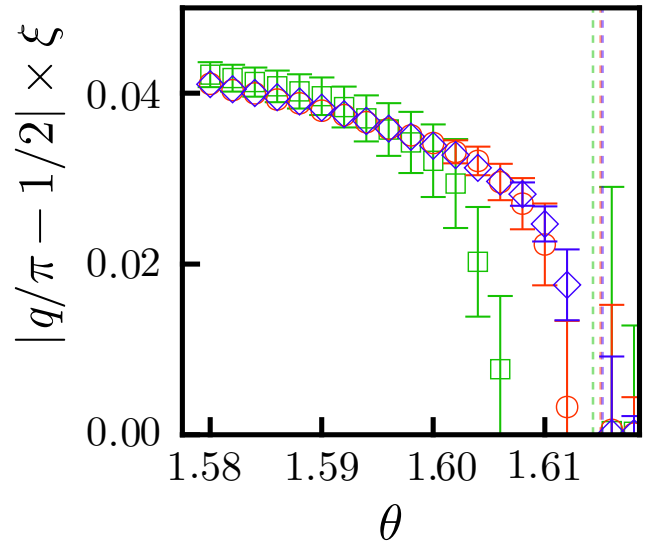




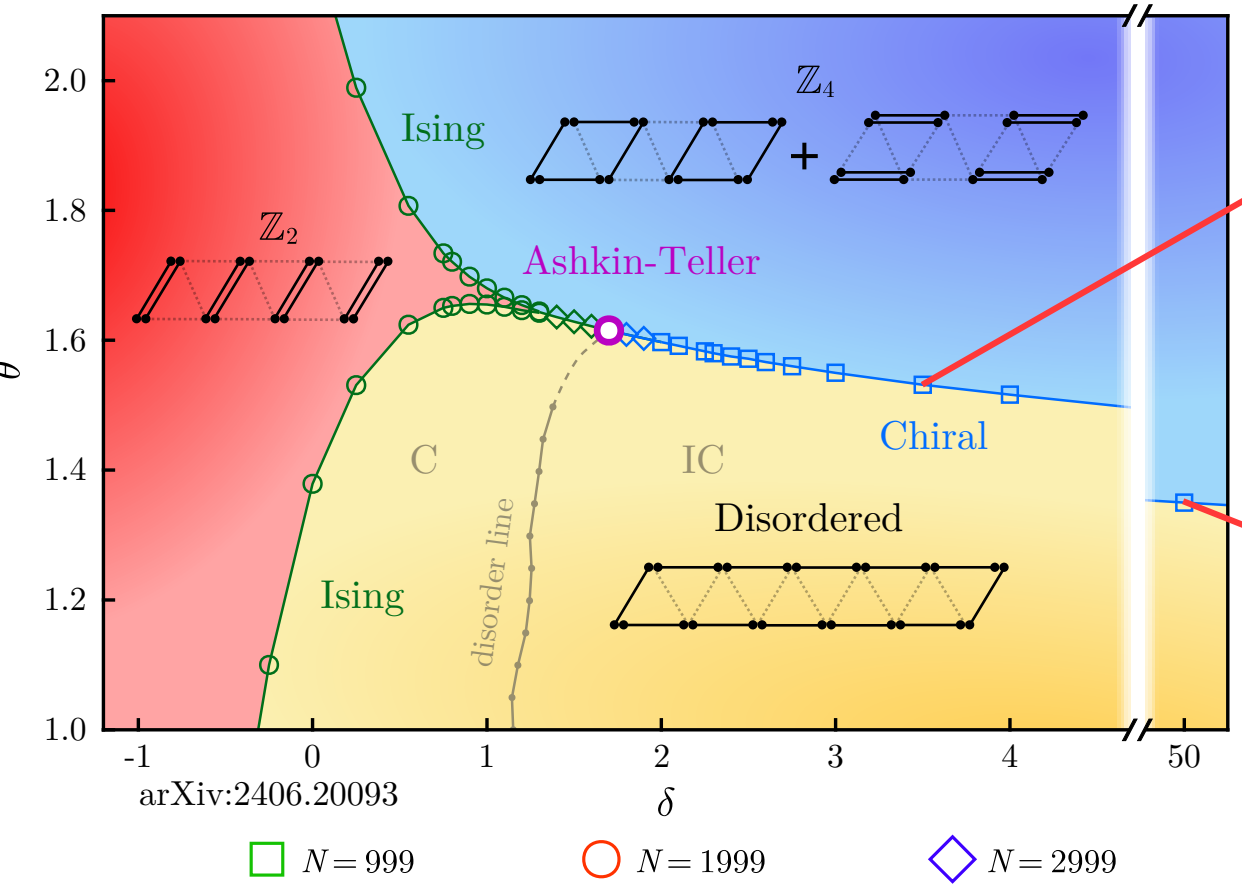
Ashkin-Teller point and chiral

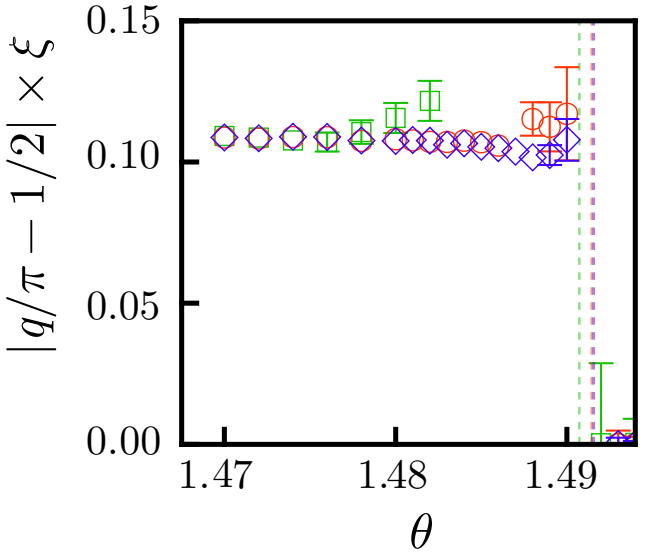


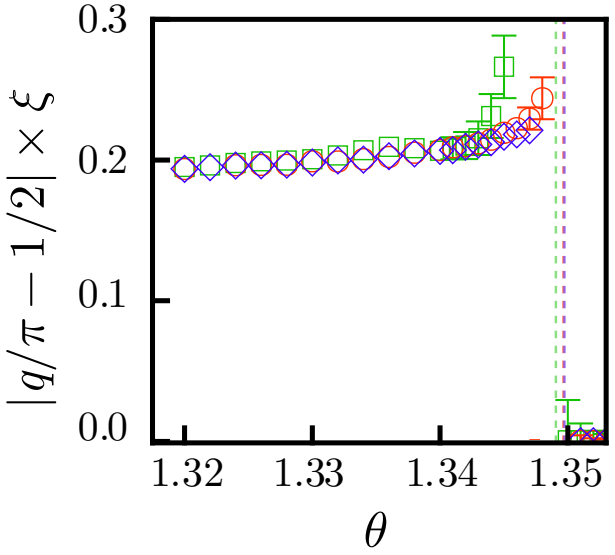




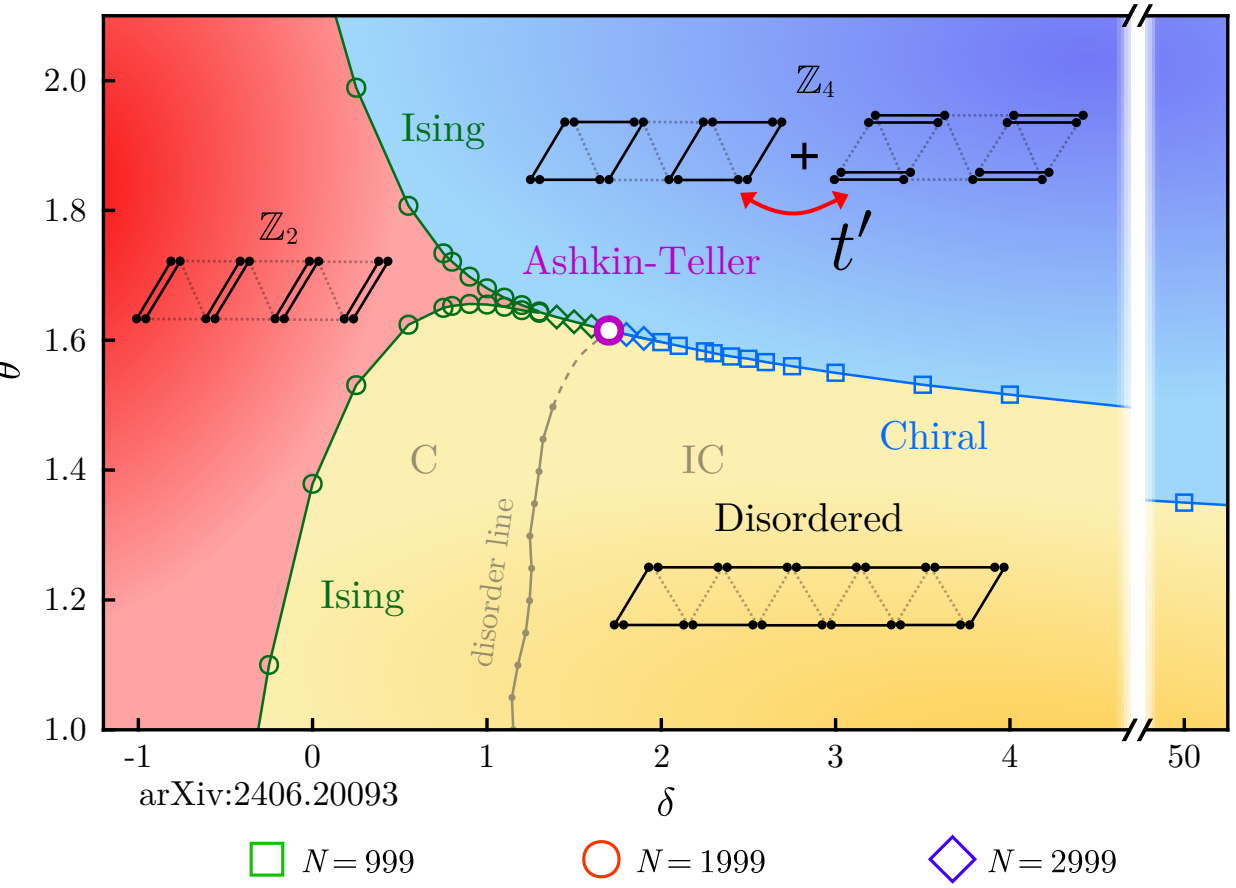
Extended chiral transition



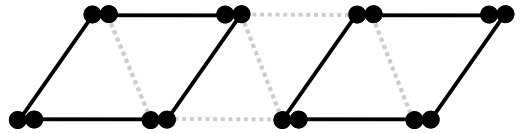




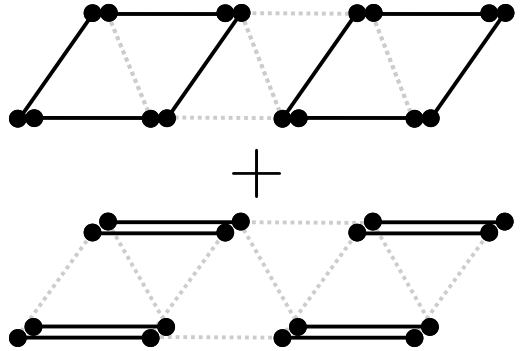
Can we manipulate this phase diagram?



$t' = 0$

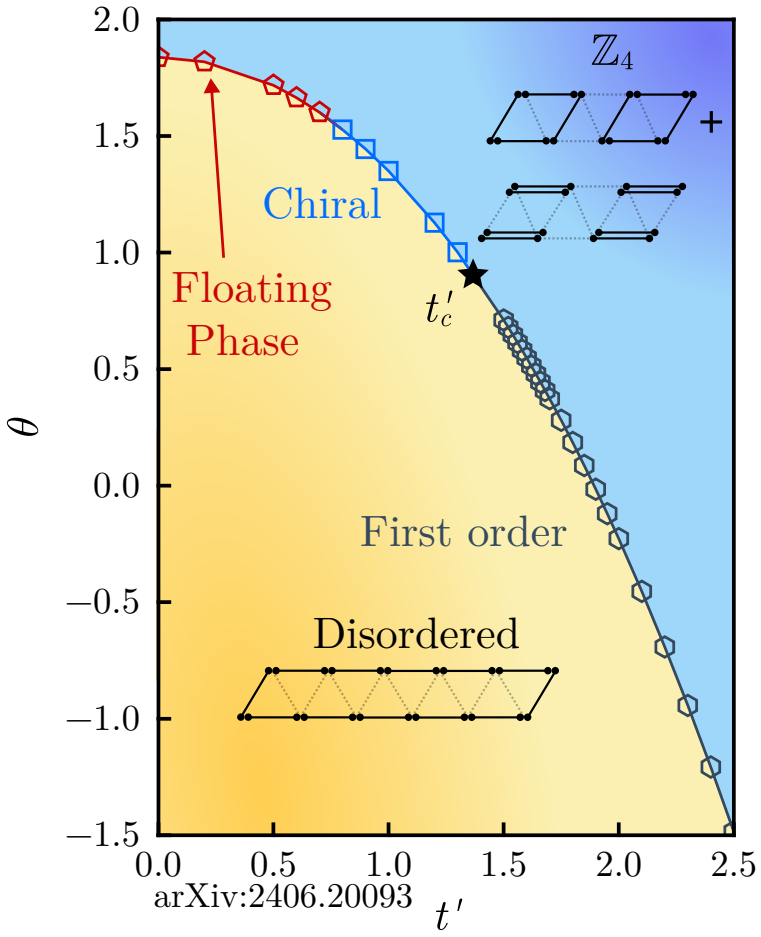


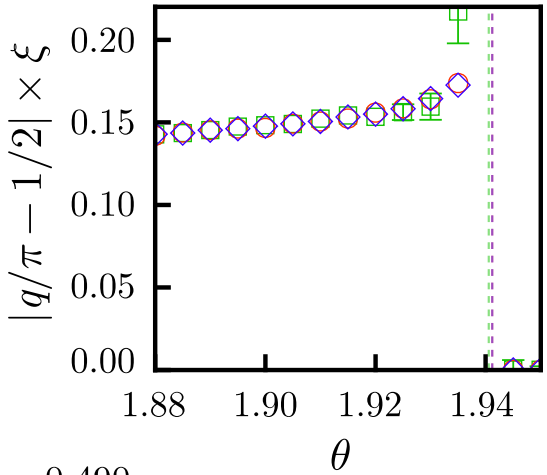
$t' > 0$

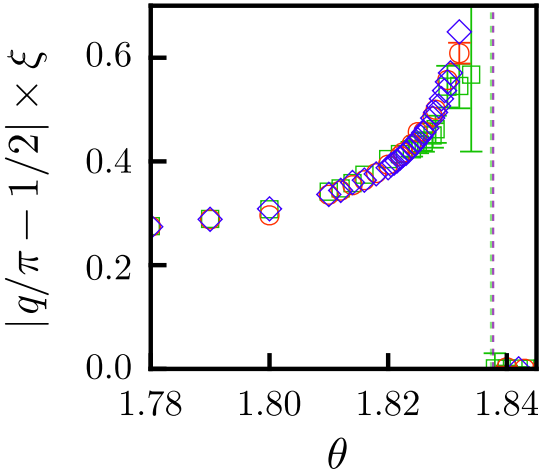


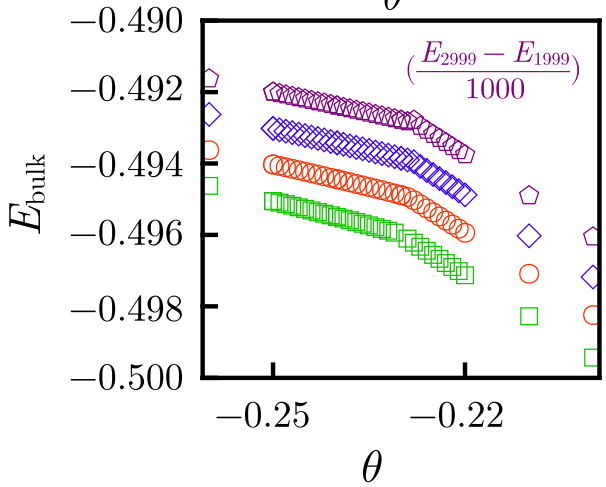
$\delta = 50$

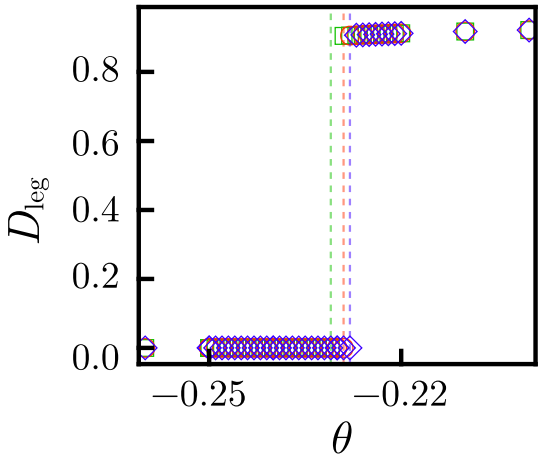
Plaquettes are essential for critical phenomena



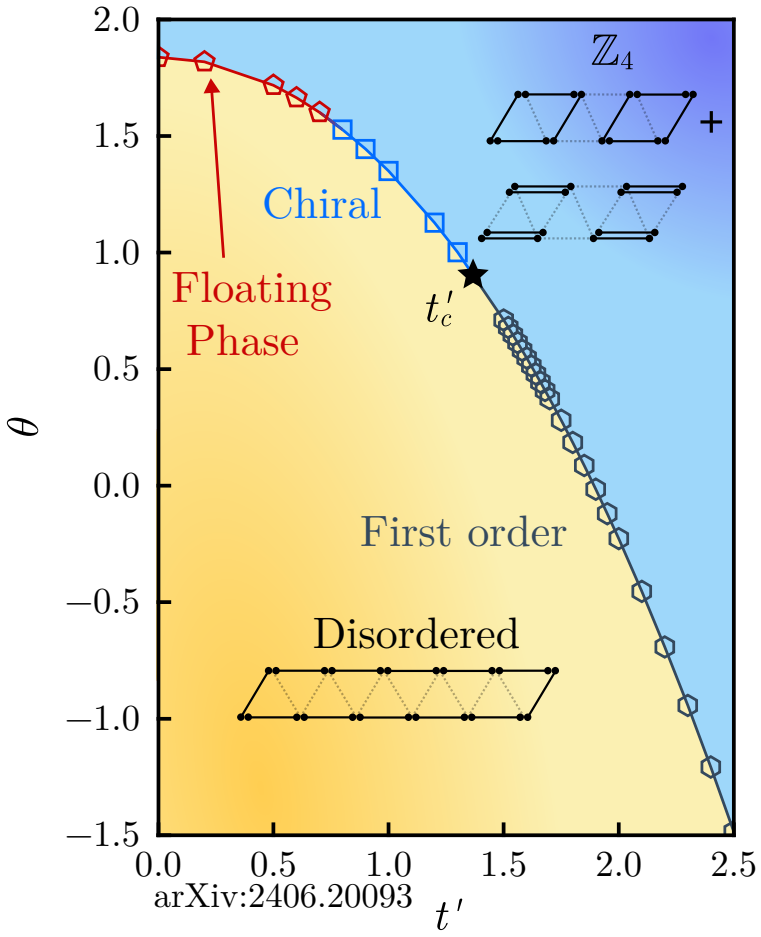




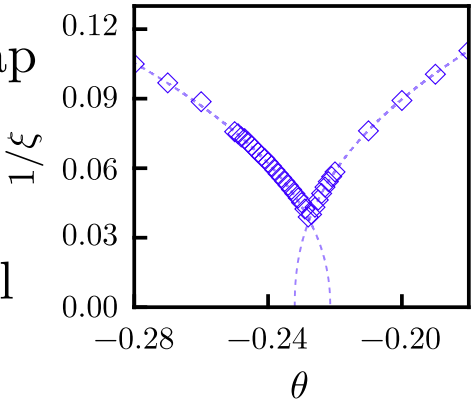


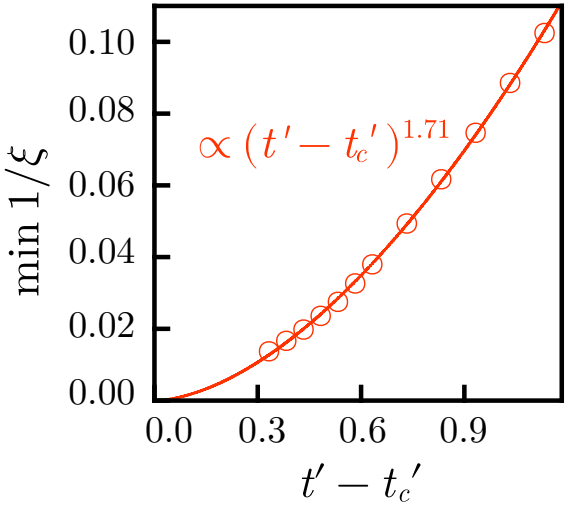


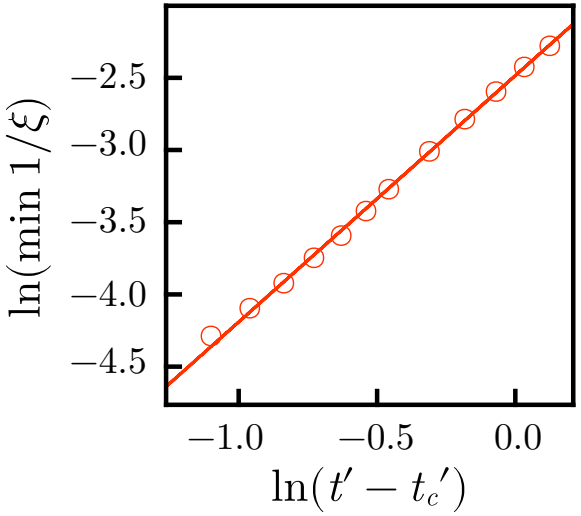
Characterizing the chiral end point

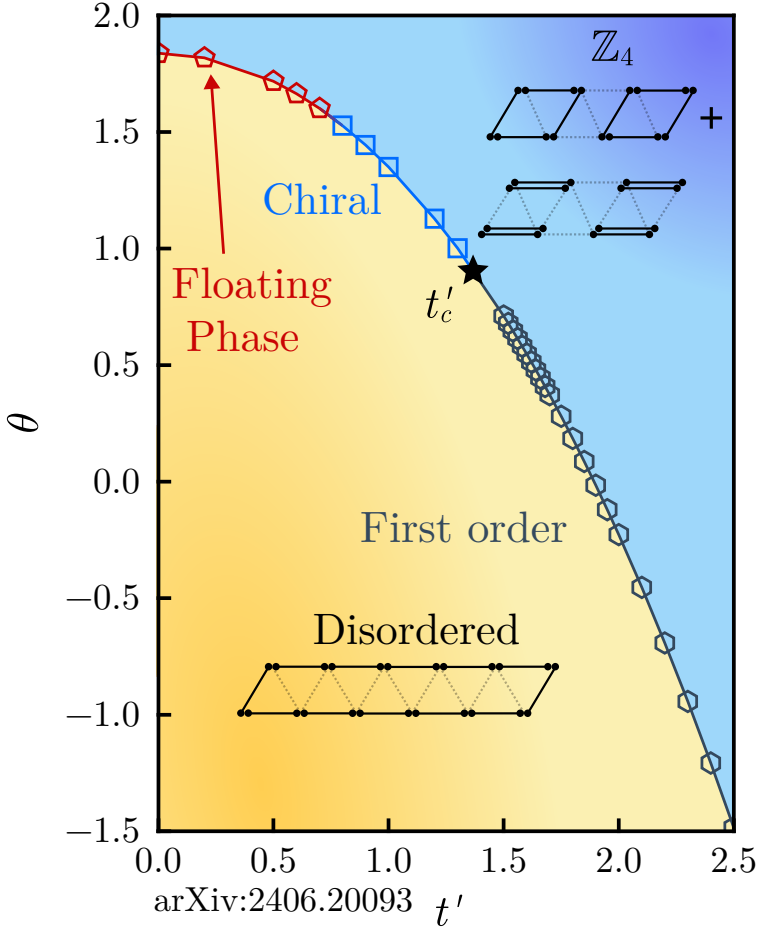


- Conformal: Scaling energy gap
- Chiral: non-conformal
- Try minimum of $1/\xi$ instead
- Algebraic → Different critical theory of chiral end point?









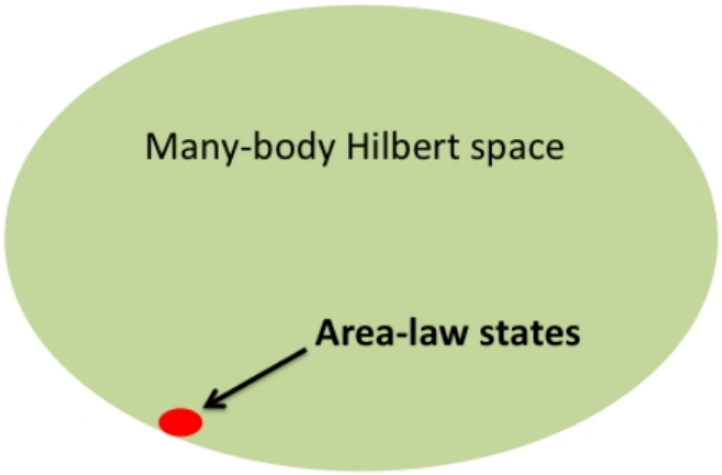
Summary

- $\mathbb{Z}_4$ transitions in QLMs on a zig-zag ladder
- Two Ising → Ashkin-Teller point
- Extended chiral transition
- Columnar density manipulates quantum criticality
- Possibly different critical theory chiral end point

Outlooks

- Nature of chiral endpoint?
- Heisenberg spin-1 chain: plaquettes essential?
- Slowly adding spins back to the model?

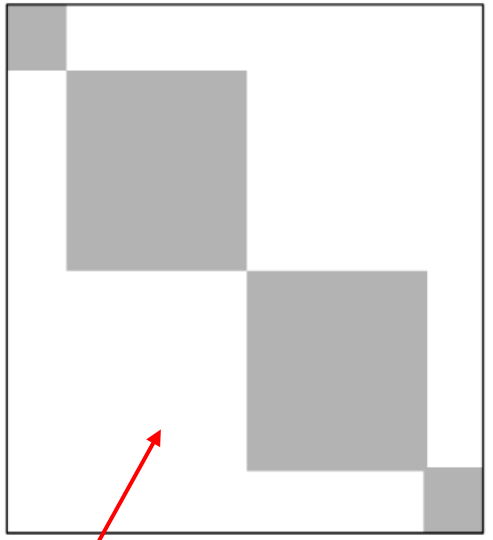
How do we simulate local 1D quantum systems?

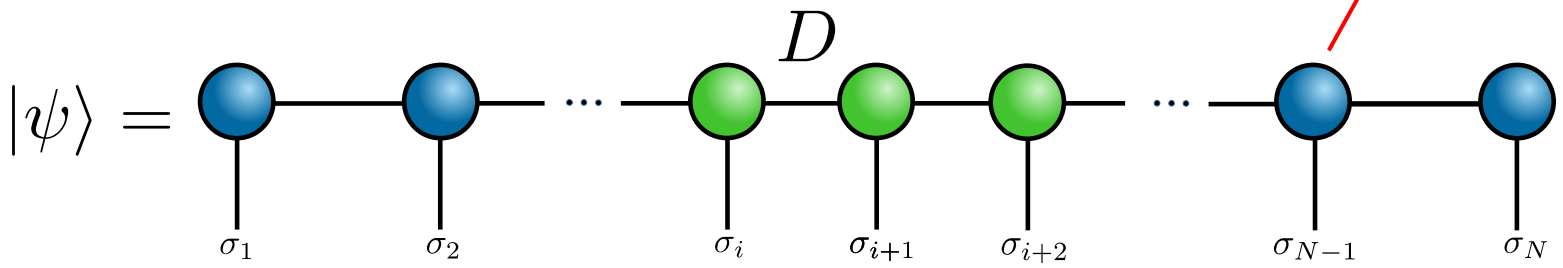


Orus, Ann. Phys. **349** 117-158 (2014)

- Local Hamiltonian
- Density Matrix Renormalization group (DMRG)
- Fragmented Hilbert space
- *Constrained* DMRG

  Chepiga, Mila, SciPost Phys **6** 033 (2019)
- Simulate $N = 3000$ sites!



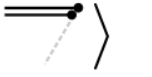


Constrained DMRG

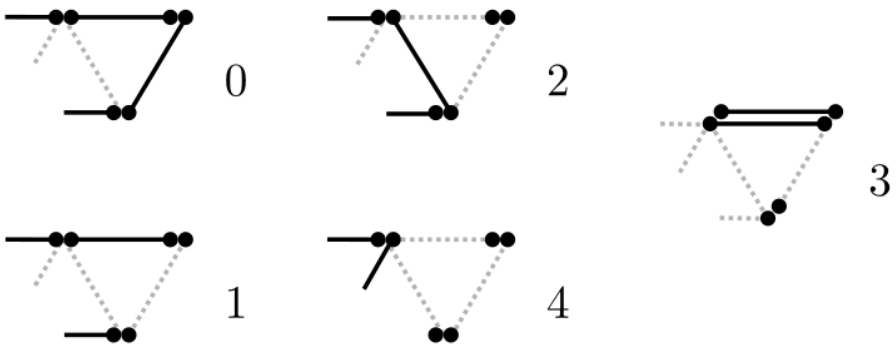
(a)

$|\psi_1\rangle =$  $|\psi_4\rangle =$ 

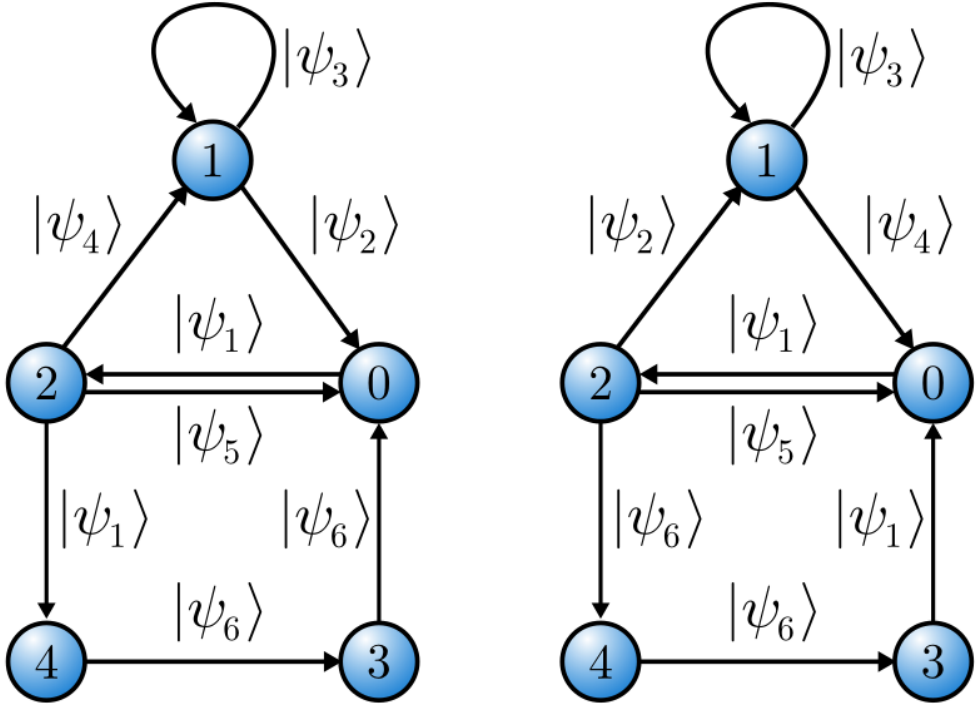
$|\psi_2\rangle =$  $|\psi_5\rangle =$ 

$|\psi_3\rangle =$  $|\psi_6\rangle =$ 

(b)

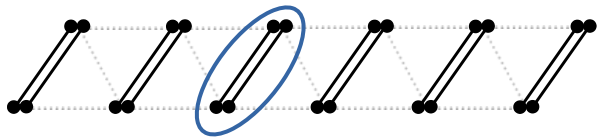


(c)

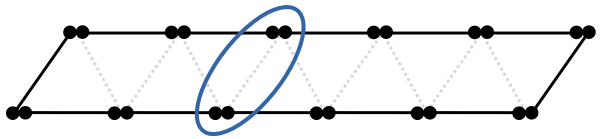


1. Ising: scaling of order parameter

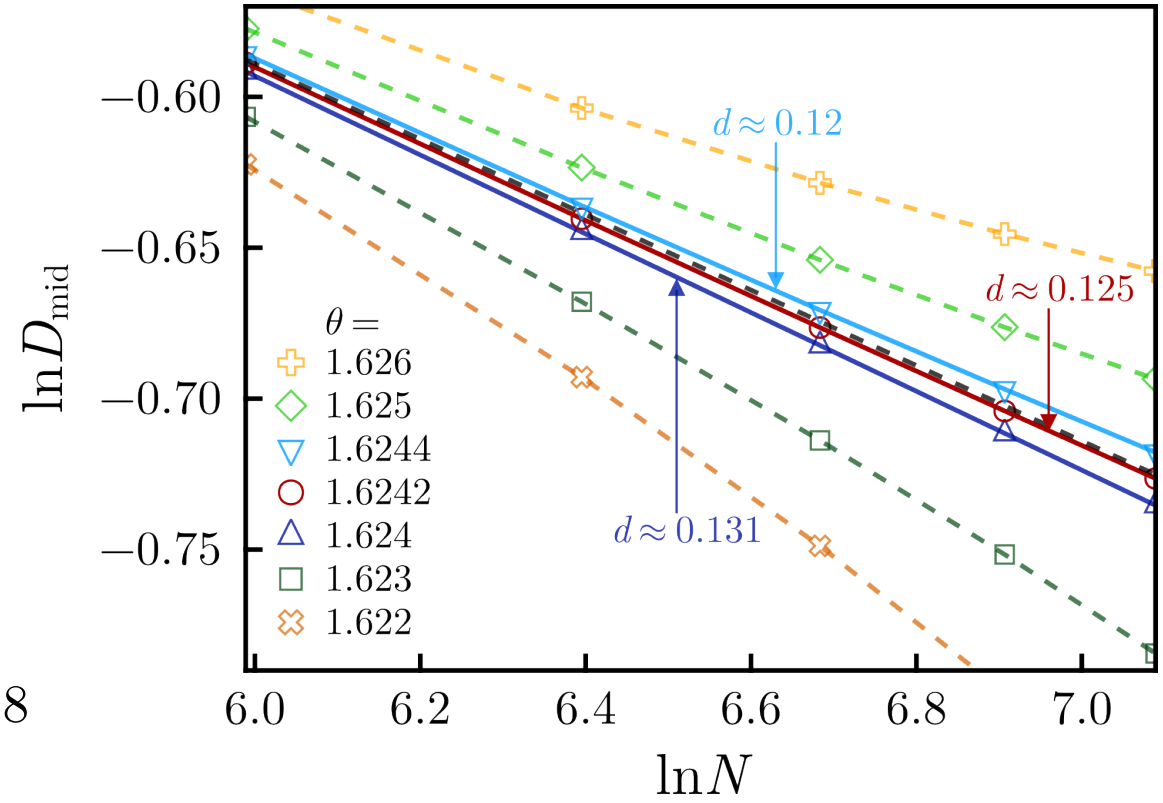
- $D_{\text{mid}} = |\langle n_i \rangle - \langle n_{i+1} \rangle|$
- versus N in log-log plot
- Convex $\to \mathbb{Z}_2$



- Concave $\to$ disorder



- Separatrix $\to$ critical
- Scaling dimension: $d = \beta/\nu = 1/8$



1. Ising: central charge

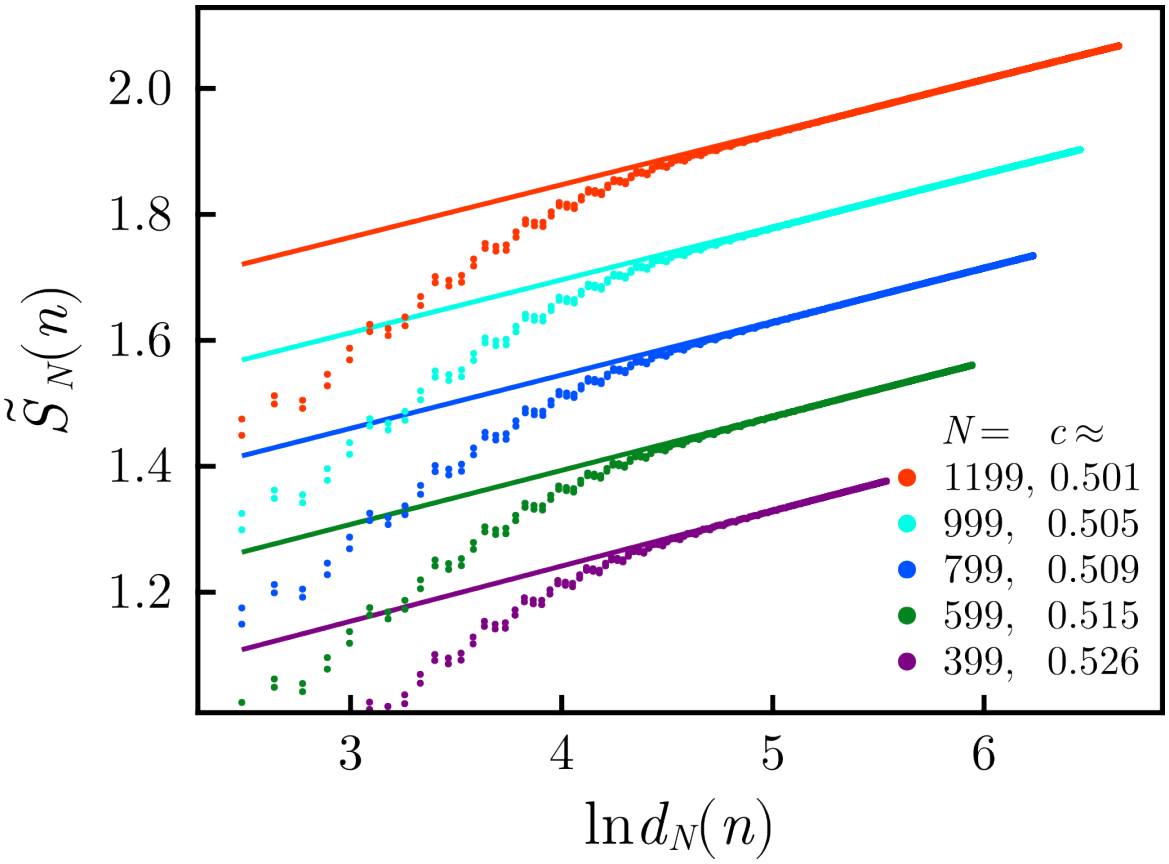
- Conformally invariant
- Central charge c
- Ising: $c = 1/2$
- Reduced entanglement entropy:

$$\tilde{S}_N(n) = \frac{c}{6} \ln d_N(n) + \ln g + s_1$$

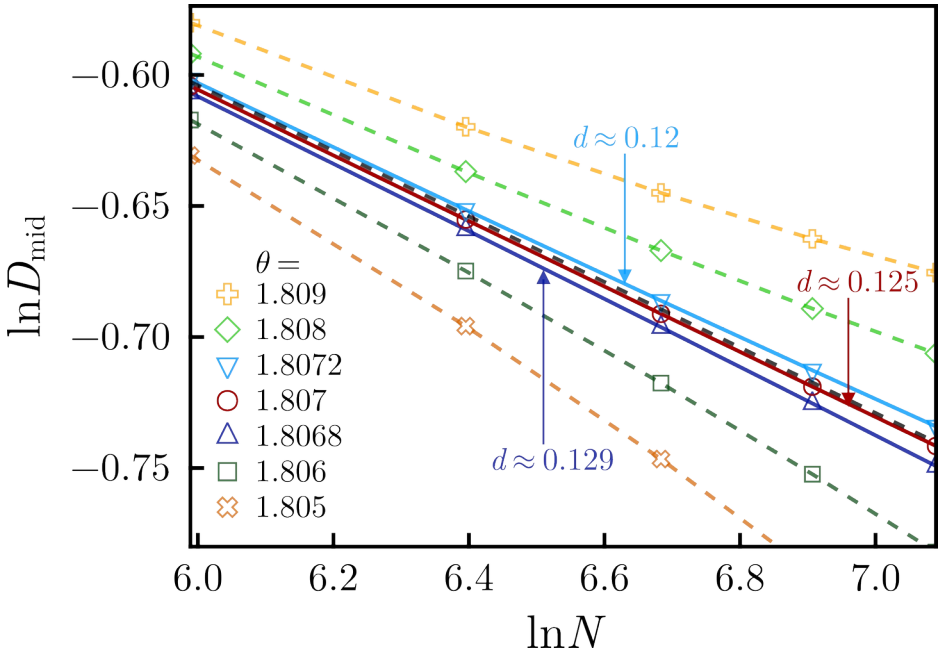
Calabrese, Cardy, J. Stat. Mech. **P06002** (2004)

- Conformal distance:

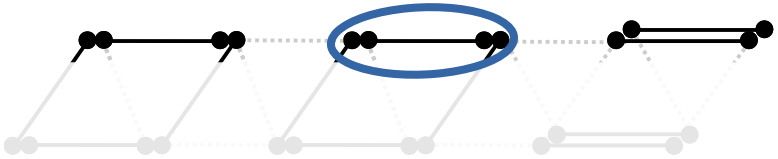
$$d_N(n) = \frac{2N}{\pi} \sin(\frac{\pi n}{N})$$

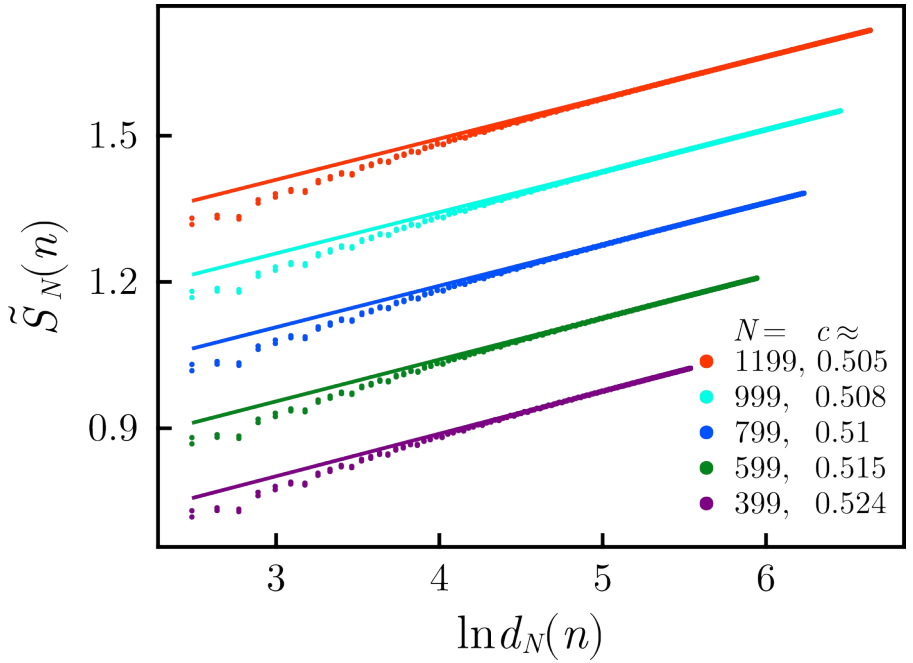


2. Ising transition between Z2 and Z4

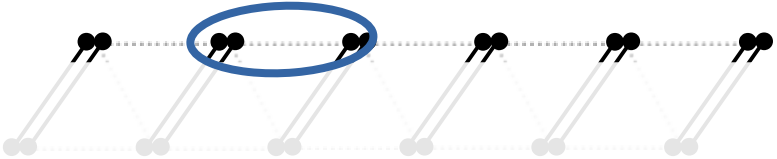


$\mathbb{Z}_4$, plaquette/columnar



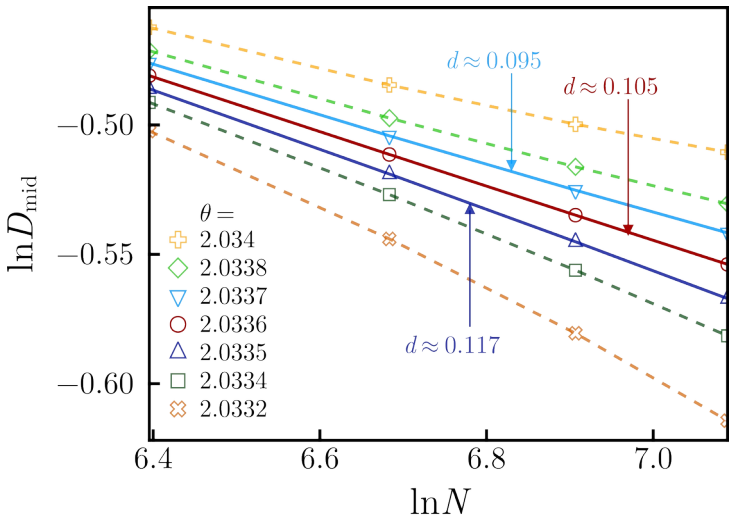


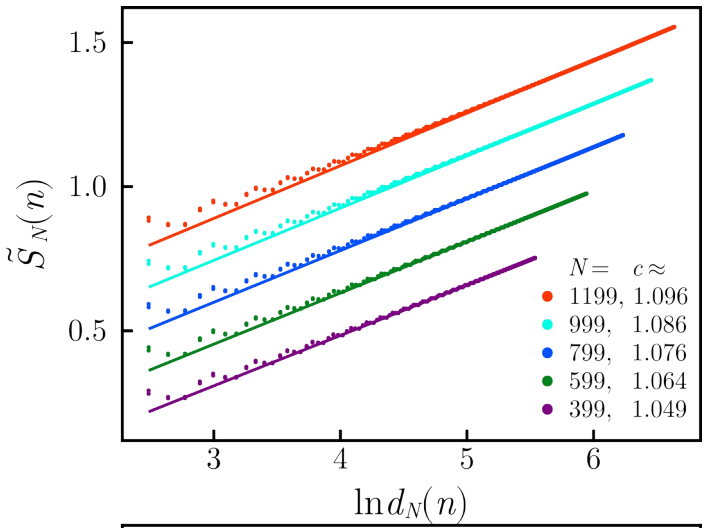
$\mathbb{Z}_2$, dimerized

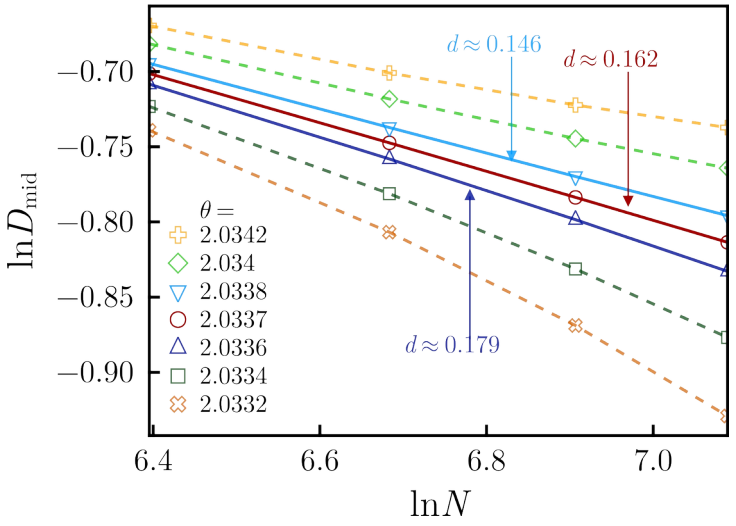


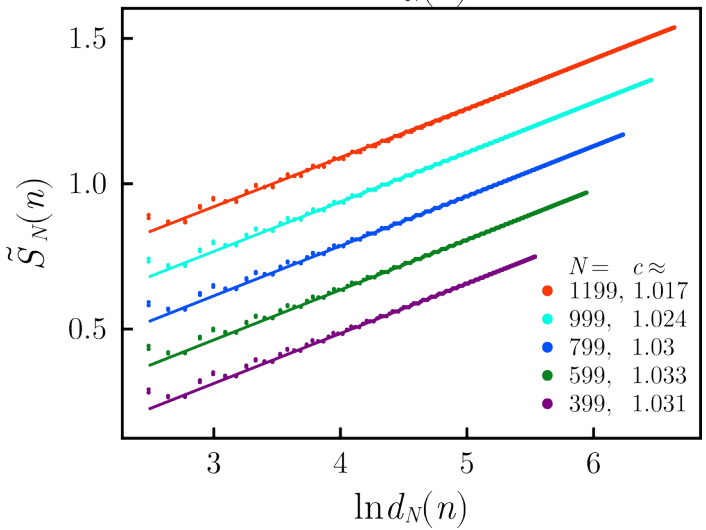
Extra data
AT point

- $d = 1/8$
- $c = 1$

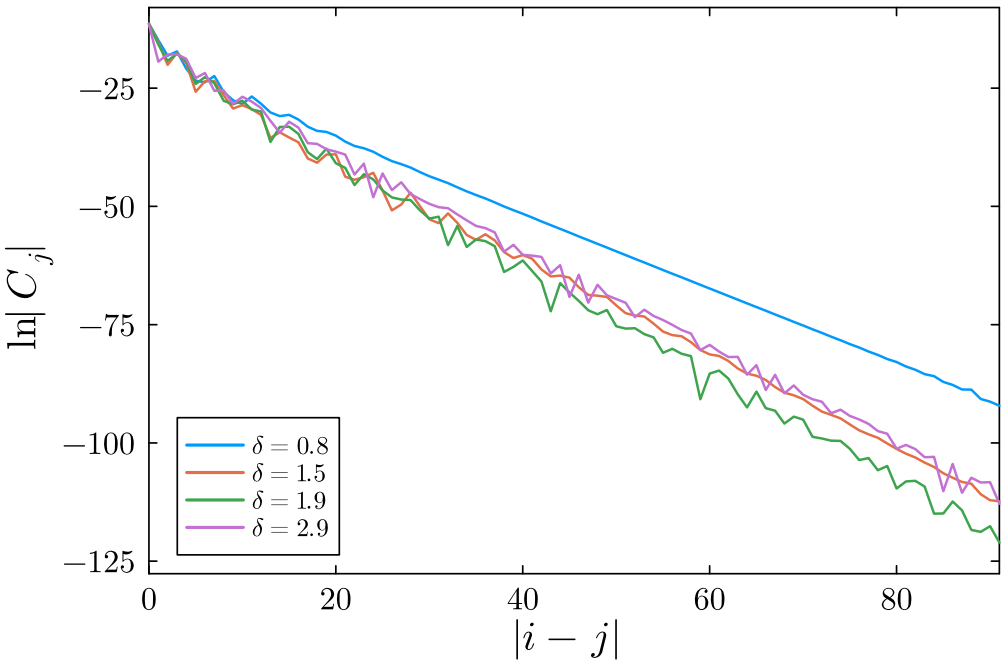


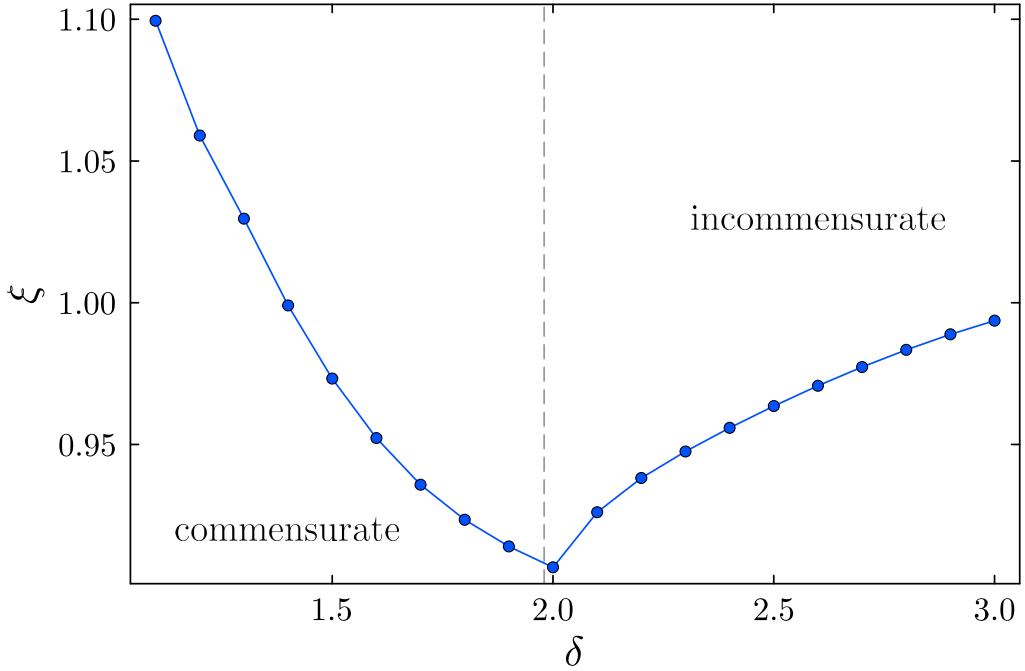






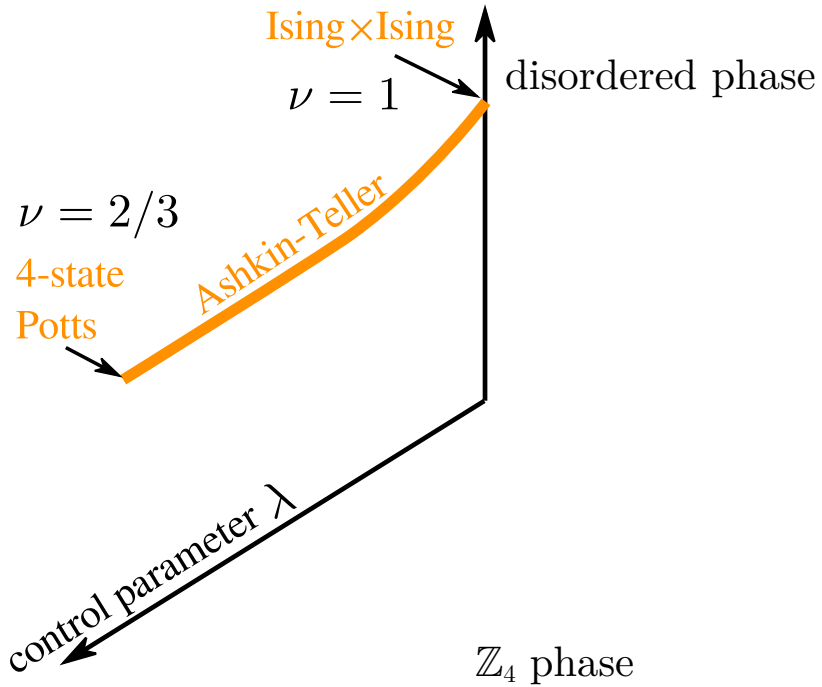
Locating the disorder line





Two Ising → Ashkin-Teller (AT)

- *Weak* universality class
- Universal d and c
- Is this the full $\mathbb{Z}_4$ transition picture?



Lüscher, Mila, Chepiga Phys. Rev. B **108** 184425 (2023)

C-IC transition

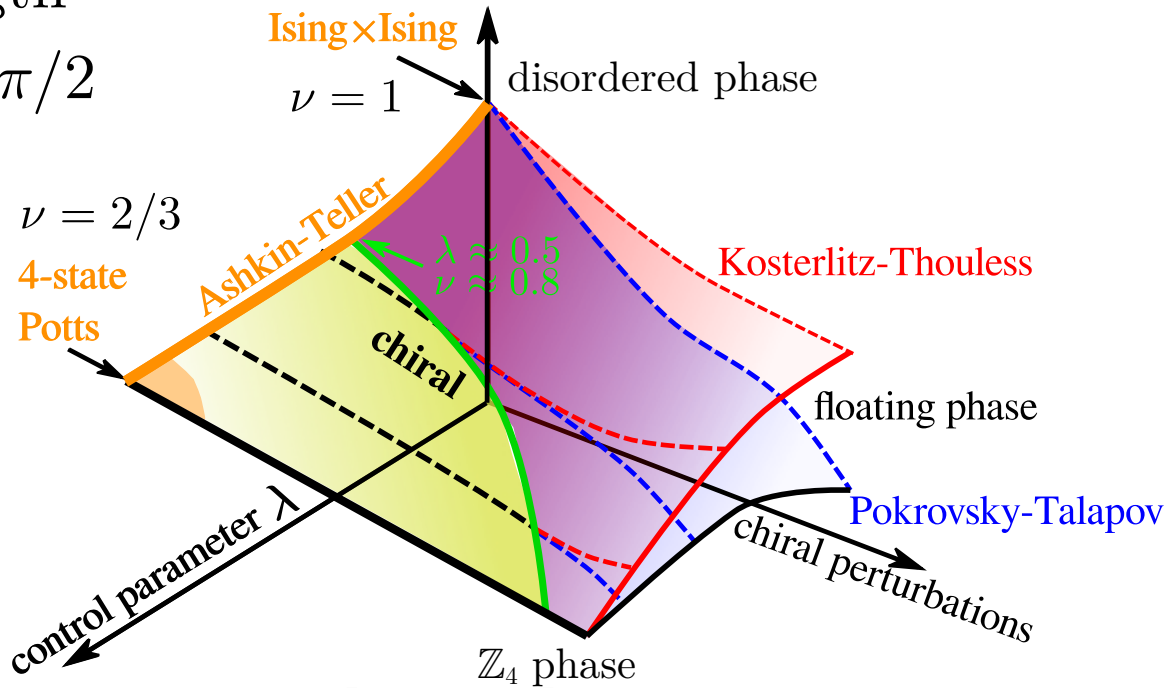
$\xi \propto |\theta - \theta_c|^{-\nu}$: correlation length

$|\Delta q| \propto |\theta - \theta_c|^{\bar{\beta}}$: distance to $q = \pi/2$

- Ashkin-Teller: $\bar{\beta} > \nu$
  $|\Delta q| \times \xi \to 0$
- Chiral: $\bar{\beta} = \nu$
  $|\Delta q| \times \xi \to$ constant
- Floating phase:
  $|\Delta q| \times \xi \to \infty$

Huse, Fisher, Phys. Rev. Lett. **49** 793 (1982)

Huse, Fisher, Phys. Rev. B **29** 293 (1984)



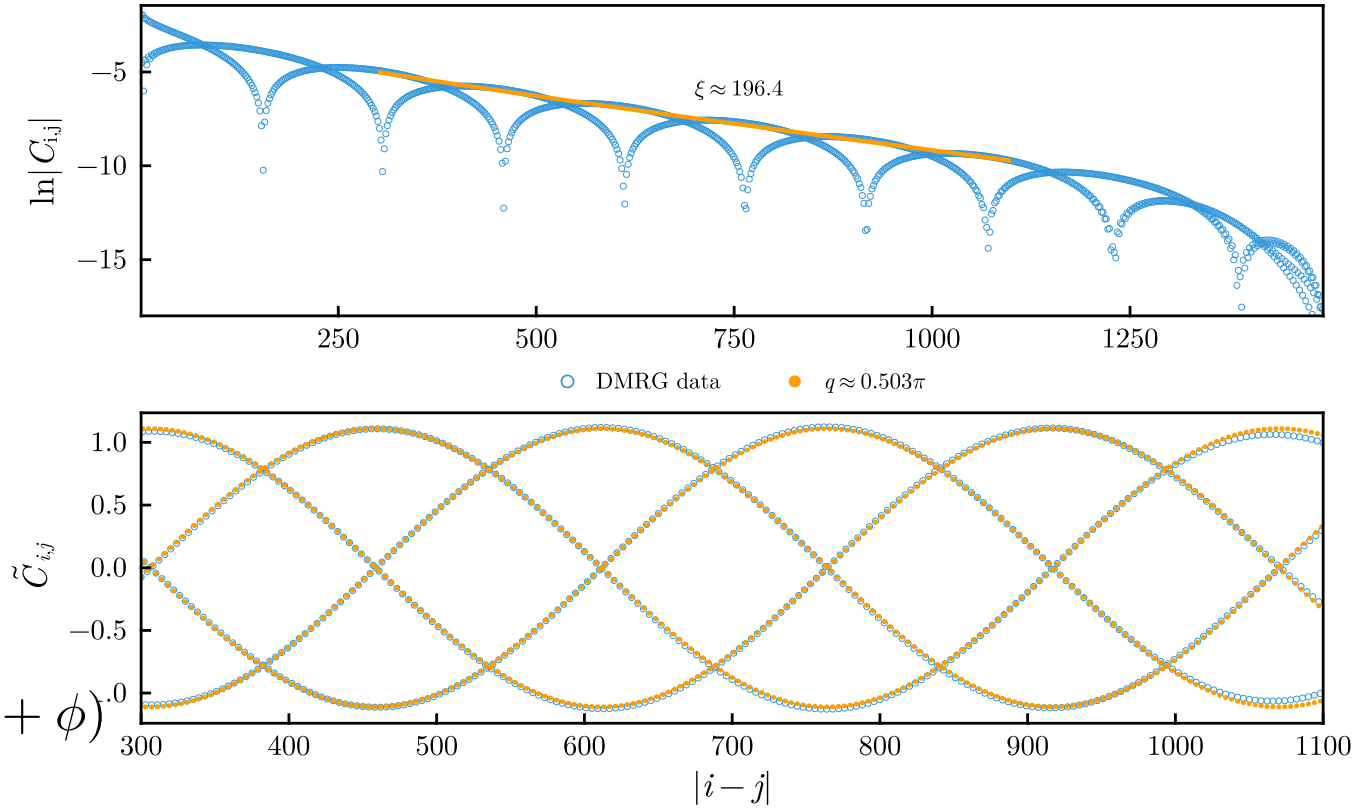
Lüscher, Mila, Chepiga Phys. Rev. B **108** 184425 (2023)

Extracting quantities

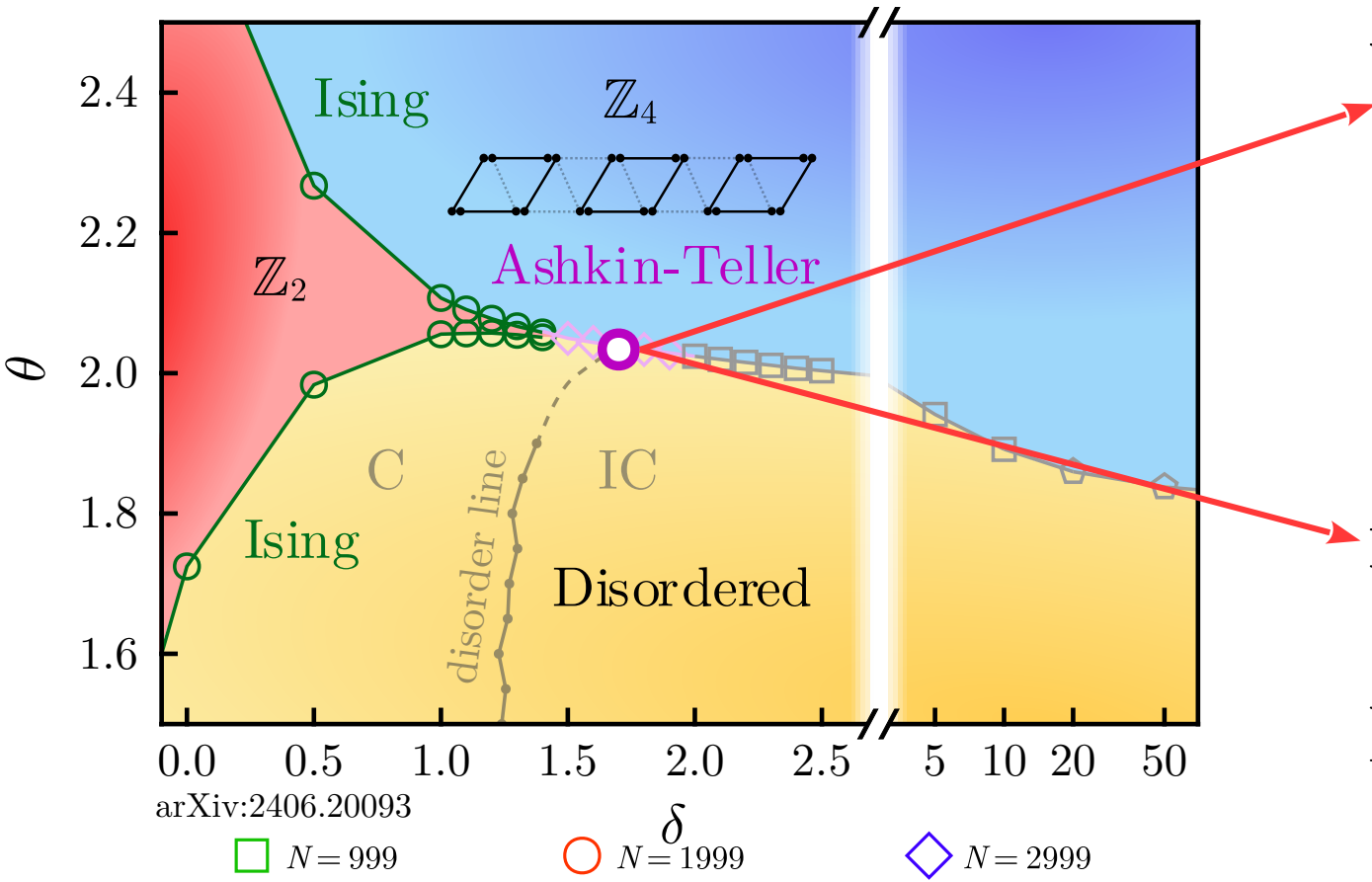
$$\xi \propto |\theta - \theta_c|^{-\nu}$$

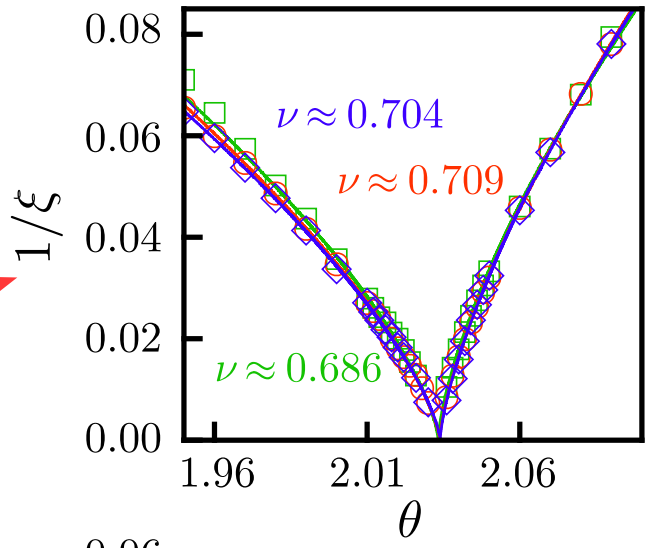
$$|\Delta q| \propto |\theta - \theta_c|^{\bar{\beta}}$$

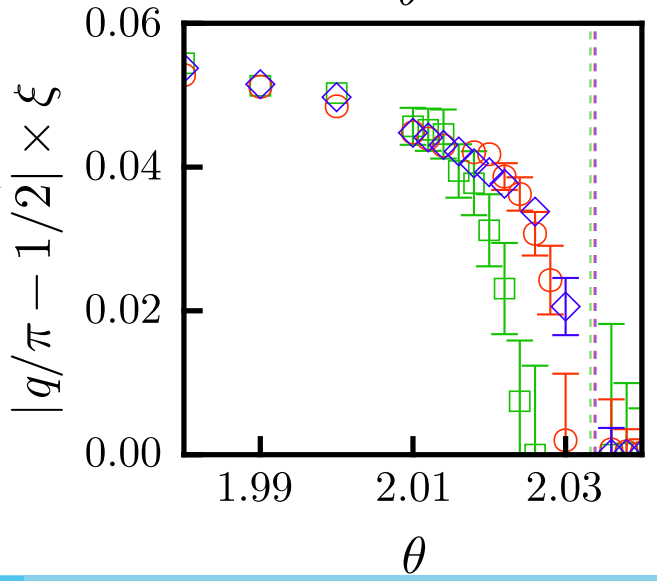
$$C_{i,j} \propto A_0 \frac{e^{-|i-j|/\xi}}{\sqrt{r}} \cos(q|i-j|\pi + \phi)$$



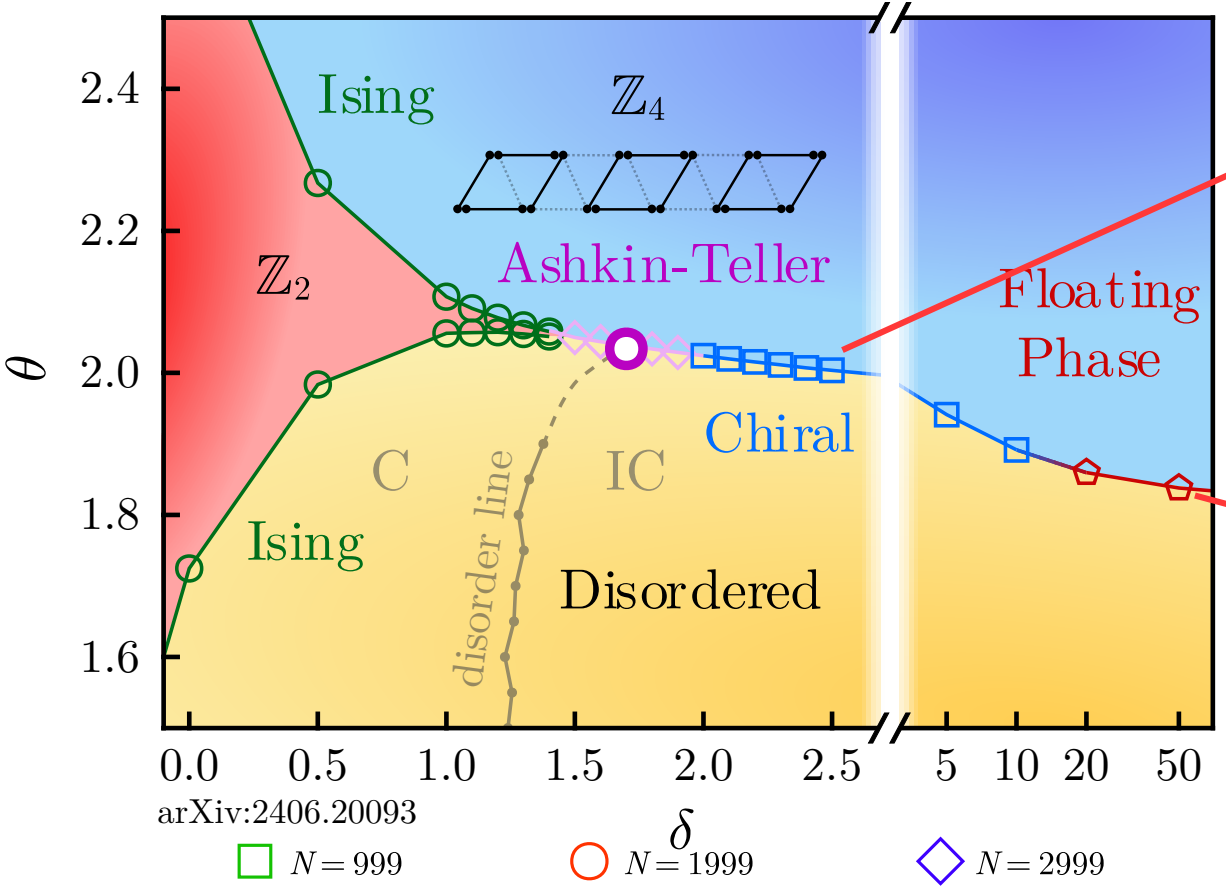
$t'=0$: Ashkin-Teller

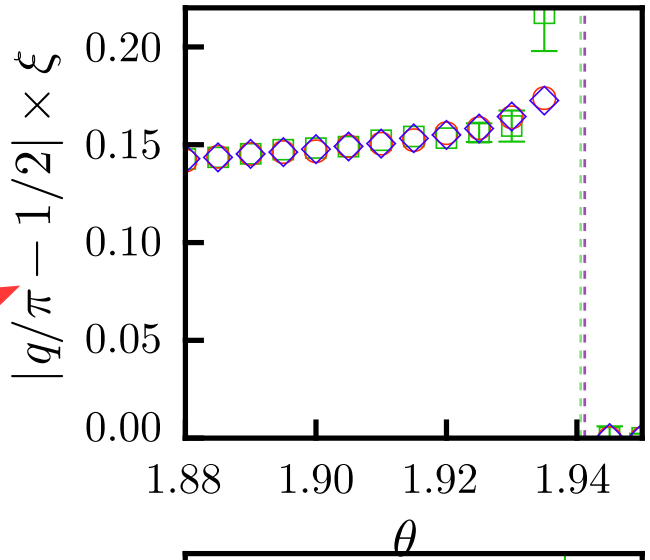


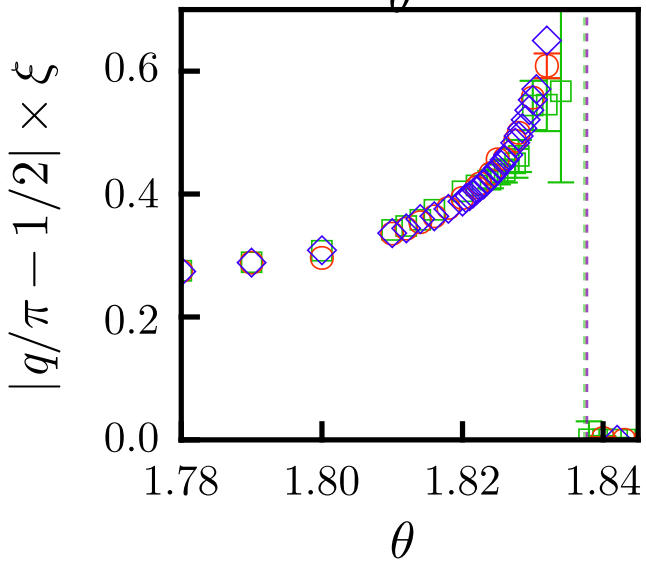




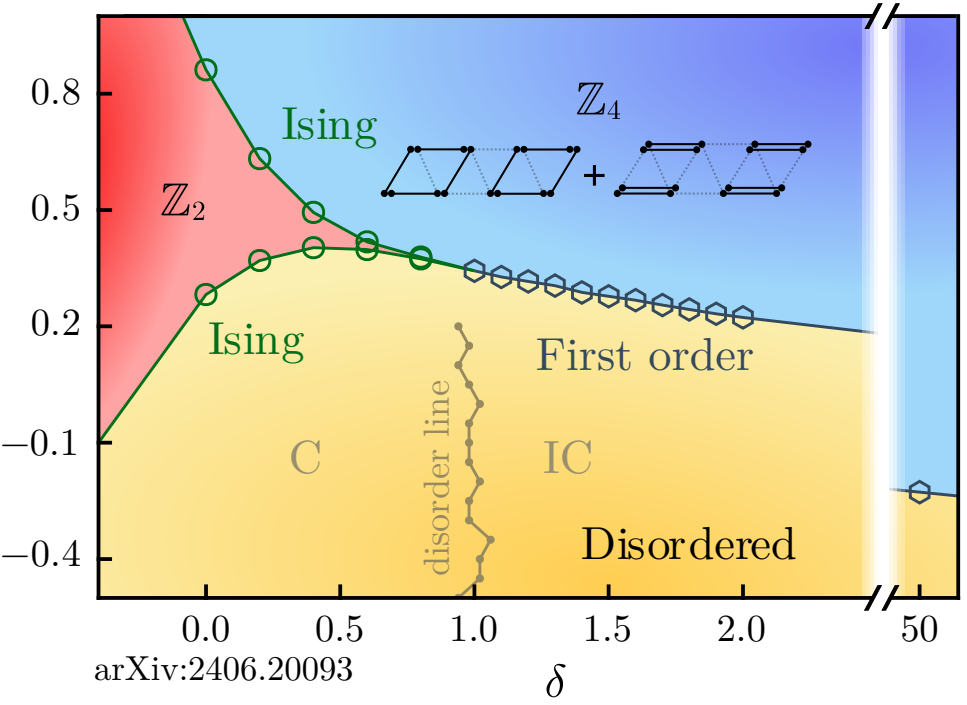
$t'=0$: Chiral & Floating phase



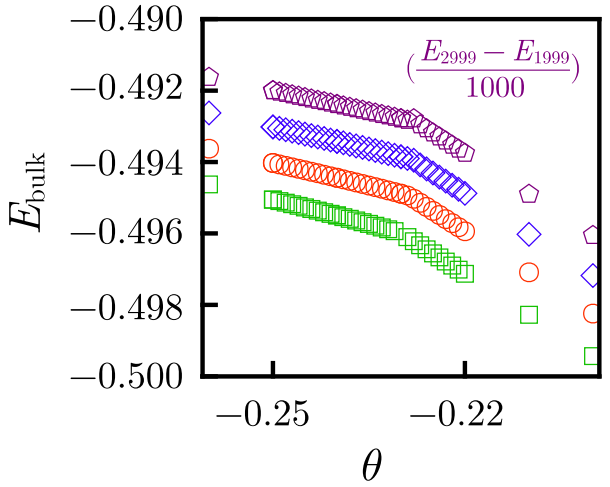


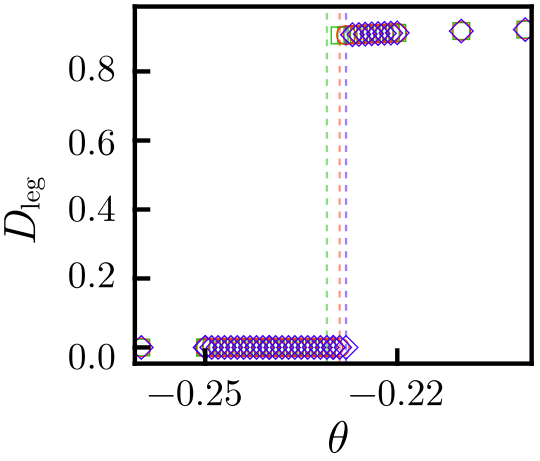


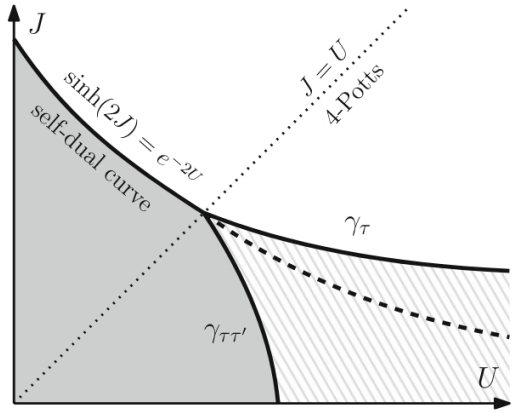
$t'=2$: First order



arXiv:2406.20093



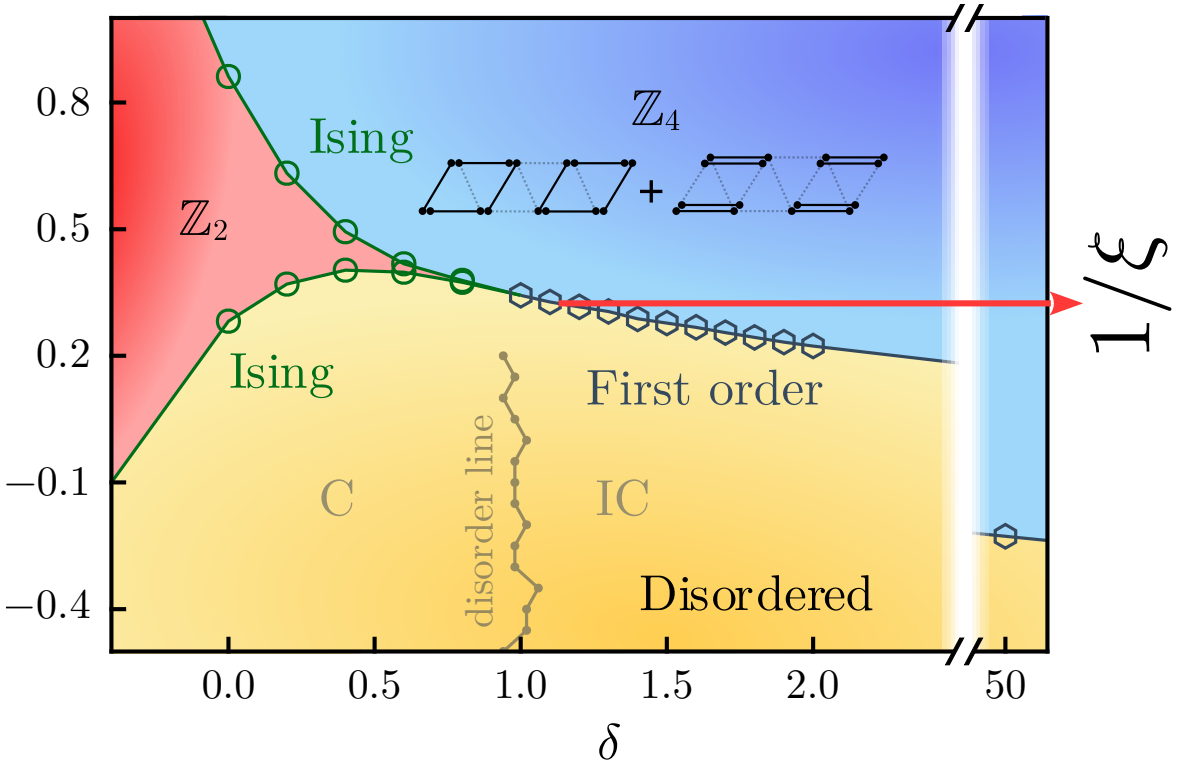


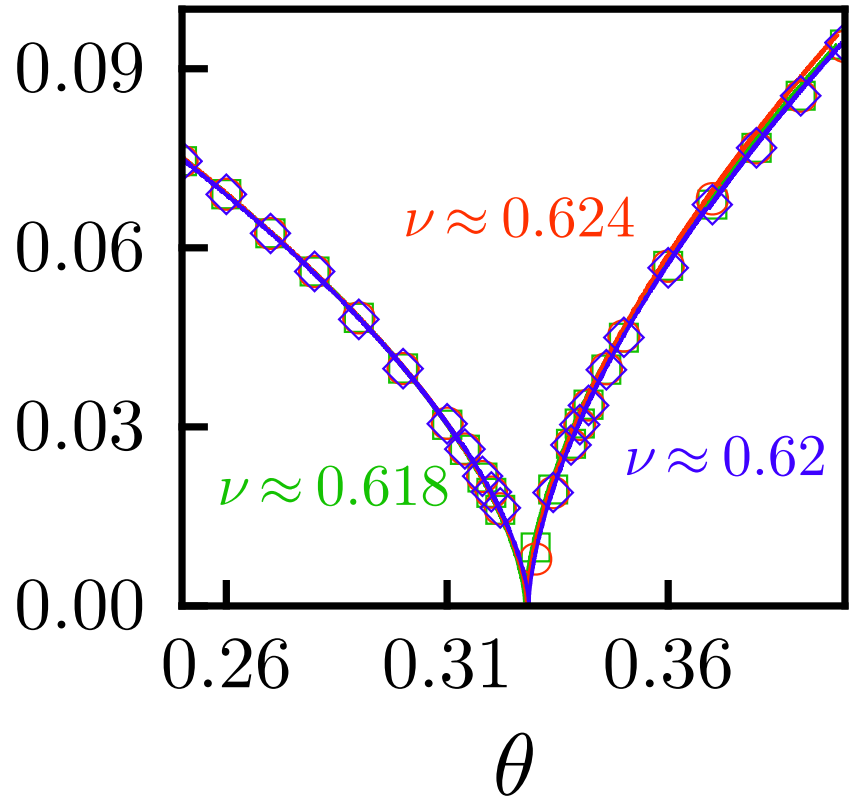


Aoun, Dober, Glazman, Commun. Math. Phys. **405** 37 (2024)

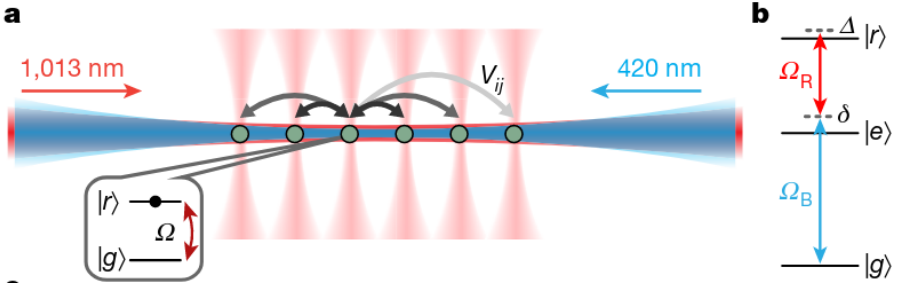
$N=999$ $N=1999$ $N=2999$

Weak first order

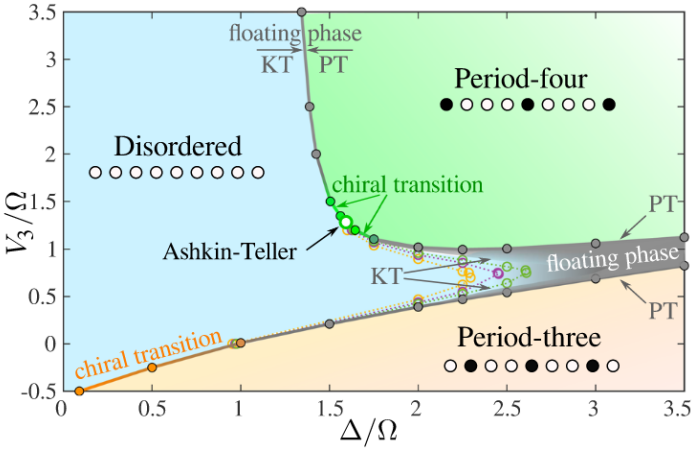




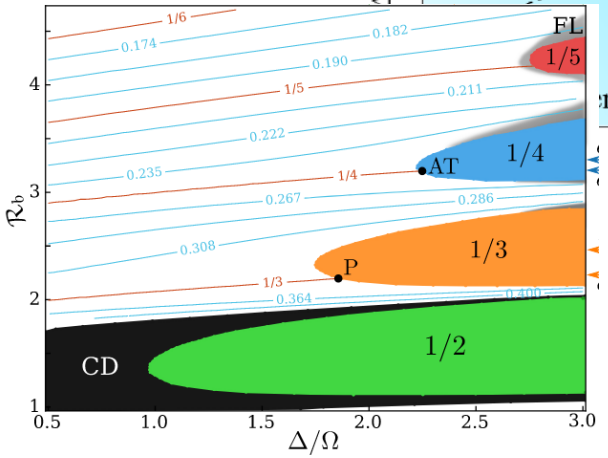
$\mathbb{Z}_4$ chiral transition in Rydberg atoms



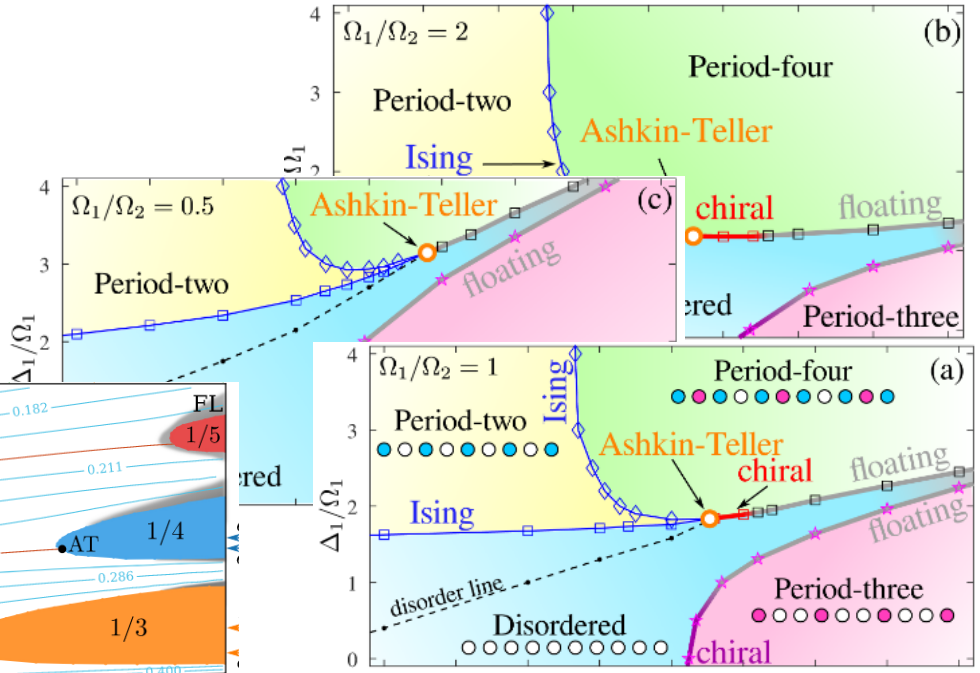
Bernien, *et al.* Nature **551** 579-584 (2017)



Chepiga, Mila, Nat. Commun. **12** 414 (2021)

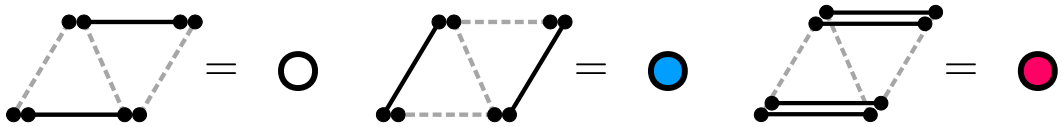


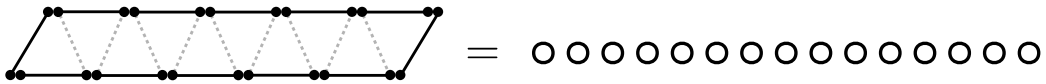
Maceira, Chepiga, Mila, Phys. Rev. Research **4** 043102 (2022)

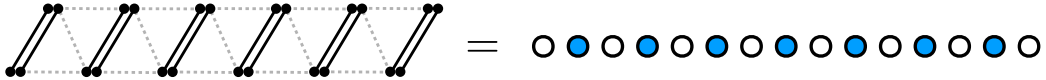


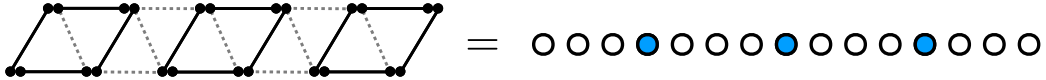
Chepiga, Phys. Rev. Lett. **132** 076505 (2024)

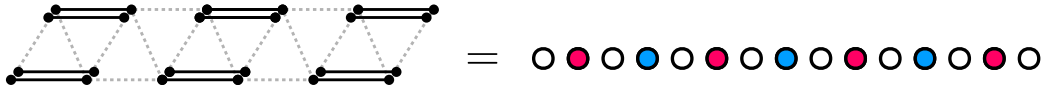
Mapping to multi-component Rydberg chains

(a) 

(b) 

(c) 

(d) 

(e) 

$$\mathcal{H}_{\text{Rydberg}} \approx \sum_i -t(c_{1,i} + c_{1,i}^\dagger) - t' n_{1,i-2}(c_{2,i} + c_{2,i}^\dagger) n_{1,i+2} + (\delta + \theta) n_{1,i} n_{1,i+2}$$
$$- \theta n_{1,i} n_{1,i+4} + \theta n_{1,i-2} n_{2,i} n_{1,i+2}.$$