Thermodynamics and entanglement entropy of the Non-Hermitian SSH model

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Outline

1. Classical, quantum and topological phase transitions

2. Hill thermodynamics

3. Non-Hermitian SSH model

4. Thermodynamics of non-Hermitian systems

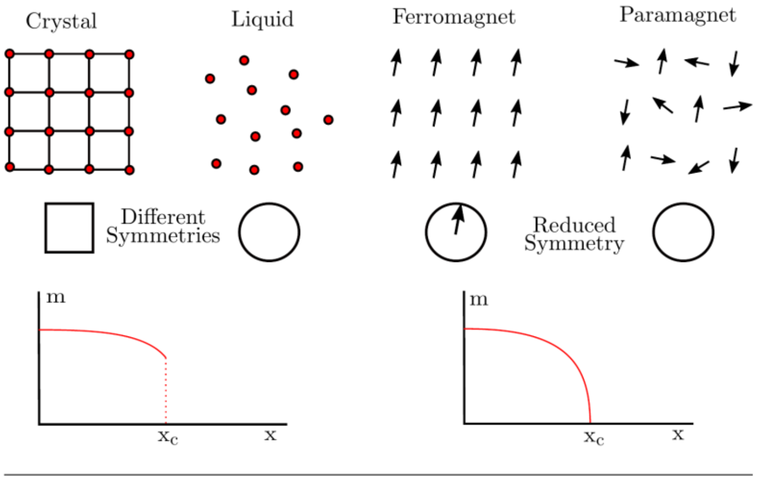
5. Conclusions and outlook

Classical, quantum and topological phase transitions

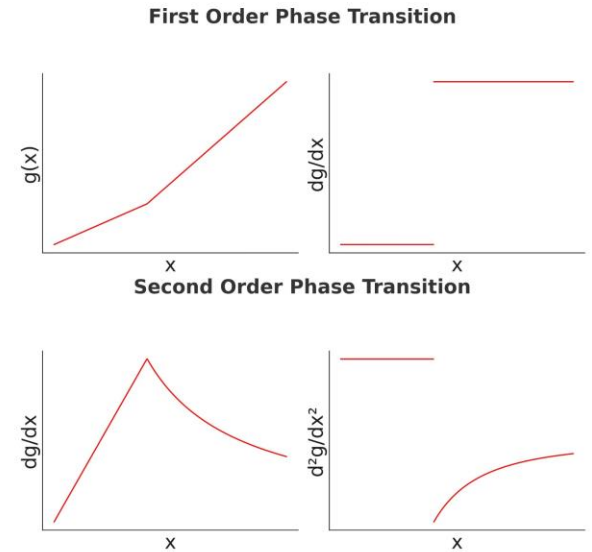
Landau classification

Discontinuous

Continuous



Ehrenfest classification



Topological phase transitions

Topological phases are not characterized by symmetry-breaking, but rather by topological invariants: transitions captured by change of invariant, non-local order parameter

What happens at finite temperature??

Hill thermodynamics

Finite size effects are important!

$$E_t = TS_t - pV_t + \mu N_t \longrightarrow dE_t = TdS_t - p\mathcal{N}dV - \hat{p}Vd\mathcal{N} + \mu dN_t$$

S_t : total entropy

$V_t = V\mathcal{N}$: total volume

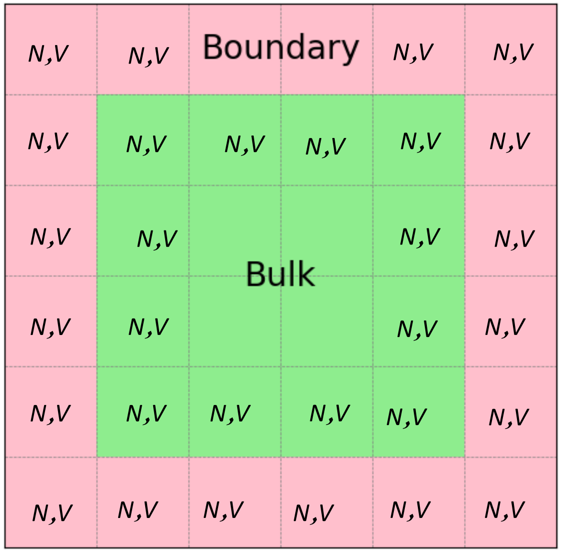
N_t: total number of particles

$\hat{p}$ is the pressure per subsystem

$-\hat{p}Vd\mathcal{N}$ work done by the subsystems

Integrating at constant V

$$E_t = TS_t + \mu N_t - \hat{p}V\mathcal{N}$$



Dividing every term by $\mathcal{N}$

$$E = TS + \mu N - \hat{p}V$$

$$E = TS + \mu N - pV + \chi$$

Subdivision potential

$$\chi = (p - \hat{p})V$$

T. L. Hill, Thermodynamics of small systems, 1st ed. (Courier Corporation, New York, 1994).

Hill thermodynamics

The partition function

$$\mathcal{Z} = \sum_{\Psi_m} \left\langle \Psi_m \middle| e^{(\hat{H}-\mu\hat{N})} \middle| \Psi_m \right\rangle$$

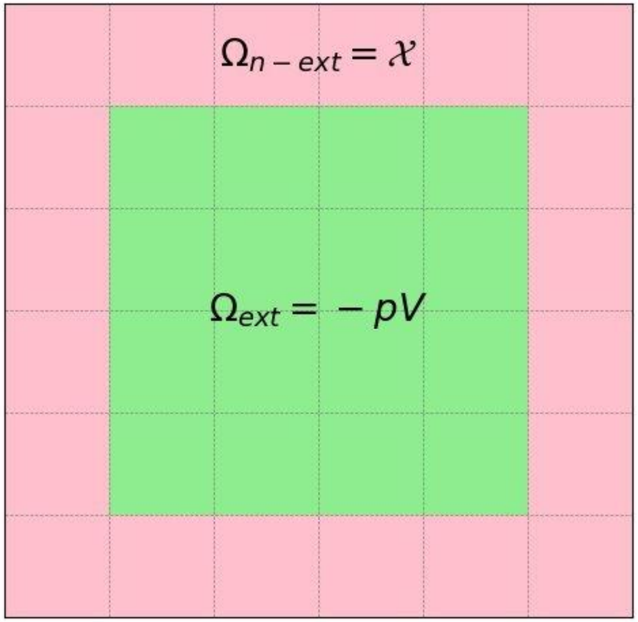
$$\mathcal{Z} = \mathrm{Tr}\{\exp(\hat{H} - \mu\hat{N})\}$$

The grand potential in the Hill thermodynamics formalism

$$\Omega = -\frac{1}{\beta} \ln \mathcal{Z}$$

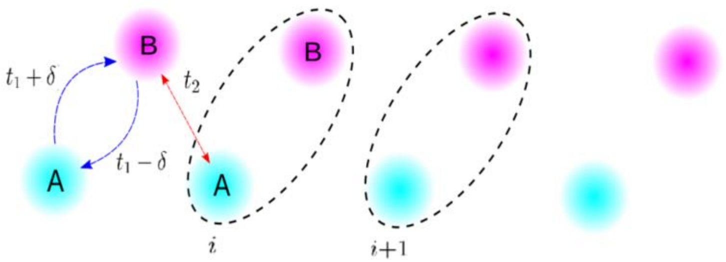
$$\Omega = E - TS - \mu N$$

$$\Omega(T = 0) = U(T = 0) = \sum_{\epsilon_n < \mu} \epsilon_n = \Omega_{ext}\, L^d + \Omega_{n-ext} L^{d-1}$$



T. L. Hill, Thermodynamics of small systems, 1st ed. (Courier Corporation, New York, 1994).

Non-Hermitian SSH model



$$H = \sum_{i=1}^{N} t_+ a_i^\dagger b_i + t_- b_i^\dagger a_i + t_2 \left(a_{i+1}^\dagger b_i + \text{h.c.} \right)$$

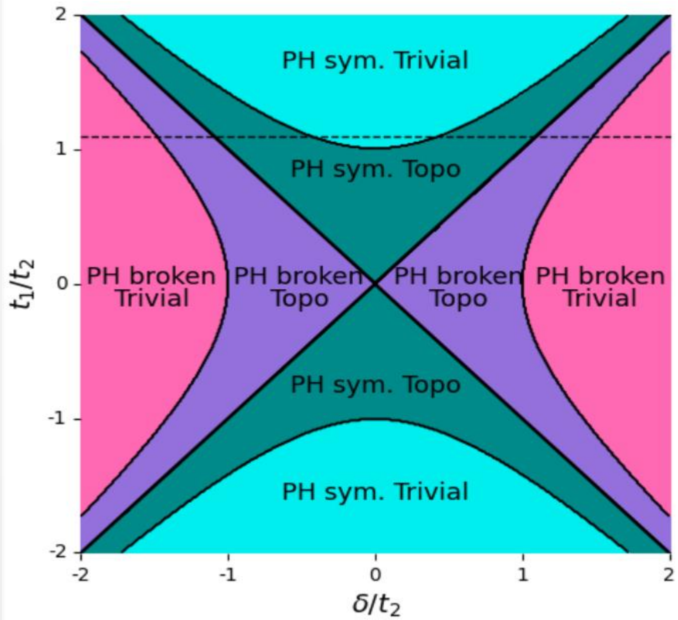
$$t_+ = t_1 + \delta \text{ and } t_- = t_1 - \delta$$

Non-Hermitian skin effect → $k \to k + i\kappa$

Surrogate Hamiltonian:

$$h_{\text{surr}}(k) = \begin{pmatrix} 0 & t_- + \sqrt{\frac{t_-}{t_+}} e^{ik} \\ t_+ + \sqrt{\frac{t_+}{t_-}} e^{-ik} & 0 \end{pmatrix} \rightarrow E_{\text{surr}\pm}(k) = \pm\sqrt{1 + t_1^2 - \delta^2 + 2\sqrt{t_1^2 - \delta^2} \cos k}$$

Phase diagram



Winding number

$$W_{\text{surr}} = \oint \frac{dk}{4i\pi} \mathbf{Tr} \left[\sigma_z h_{surr}^{-1}(k) \frac{dh_{surr}(k)}{dk} \right]$$

K. Kawabata, K. Shiozaki, M. Ueda, and M. Sato, Phys. Rev. X 9, 041015 (2019)
R. Arouca, C. H. Lee, and C. M. Smith, Phys. Rev. B. 102, 245145 (2020)

Thermodynamics of non-Hermitian systems

Thermodynamic description of intrisically open system

Biorthogonal basis $\langle \Psi_n^L | \Psi_m^R \rangle = \delta_{nm}$

$$\begin{cases} H|\Psi_m\rangle^R = \epsilon_m |\Psi_m\rangle^R \\ H^\dagger |\Psi_m\rangle^L = \epsilon_m^* |\Psi_m\rangle^L \end{cases}$$

→ *Complex thermodynamic potentials? No!!*

Pseudo-Hermitian symmetry:

Energies come in complex conjugated pairs

$$H = \eta H^\dagger \eta^{-1}$$

$$\eta = \sum_m |\Psi_m\rangle^{LL}\langle \Psi_m|$$

$$\mathcal{Z} = \sum_{\Psi_m} {}^L\langle \Psi_m | e^{(\hat{H} - \mu \hat{N})} | \Psi_m \rangle^R$$

$$\mathcal{Z} = \mathrm{Tr}\{\exp(\hat{H} - \mu \hat{N})\}$$

Entanglement entropy of the nH-SSH model

Correlation functions

$$C_{ij}^{LR} = \langle G^L | \, \phi_i^\dagger \phi_j \, | G^R \rangle$$

ξ_α : correlation eigenvalues

Entanglement entropy

$$\mathcal{EE} = -\sum_\alpha \left[\xi_\alpha \ln \xi_\alpha + (1-\xi_\alpha) \ln (1-\xi_\alpha) \right]$$

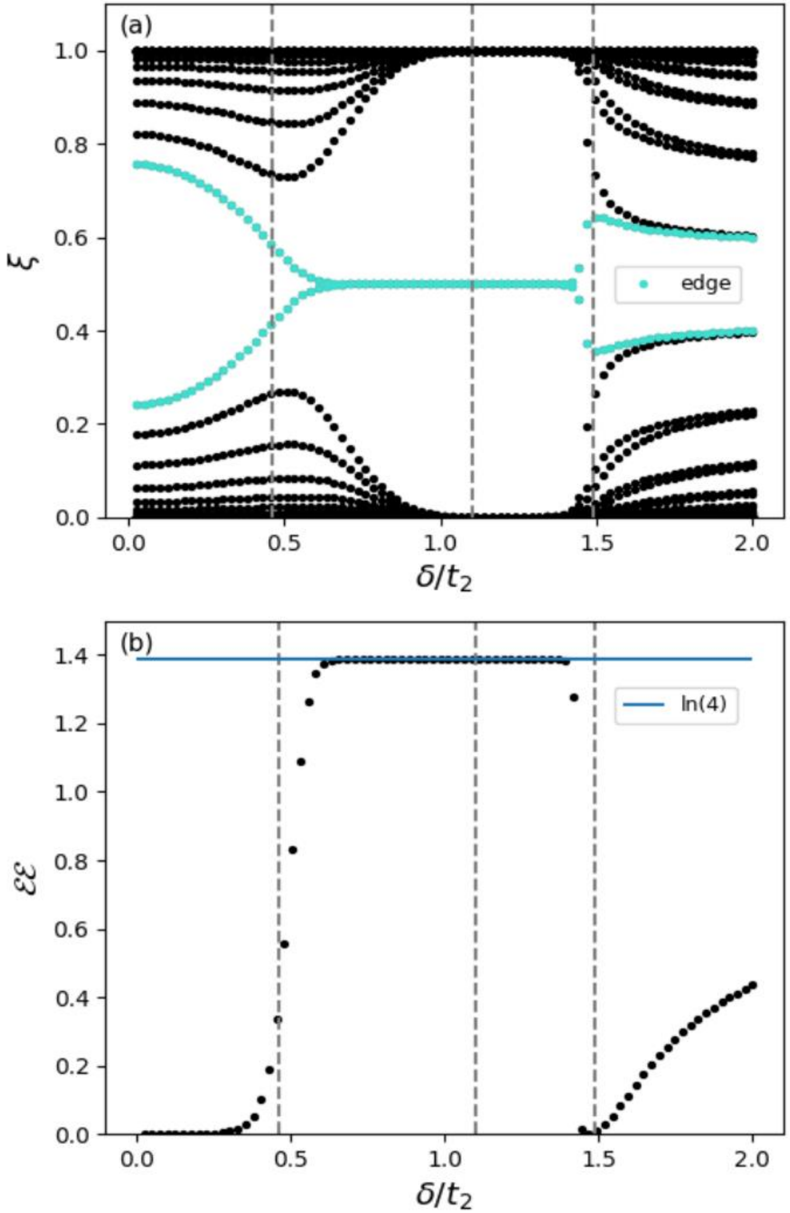
$\delta/t_2 < 0.46$ edge states are gaped ($\mathcal{EE}$=0)

At $\delta/t_2 = 0.46$ $\mathcal{EE}$ abruptly jumps to $ln(4)$
Transition from a trivial to a topological phase

$0.46 < \delta/t_2 < 1.49$ gap closes (Topological phase)

At $\delta/t_2 = 1.49$ $\mathcal{EE}$ abruptly jumps to zero
Transition from a topological to a trivial phase

$\delta/t_2 > 1.49$ reappearance of a gap (trivial phase)



Entanglement entropy of the nH-SSH model

Logarithmic scaling:

$$\mathcal{EE} \sim (c/3)\ln(L_{\mathcal{A}})$$

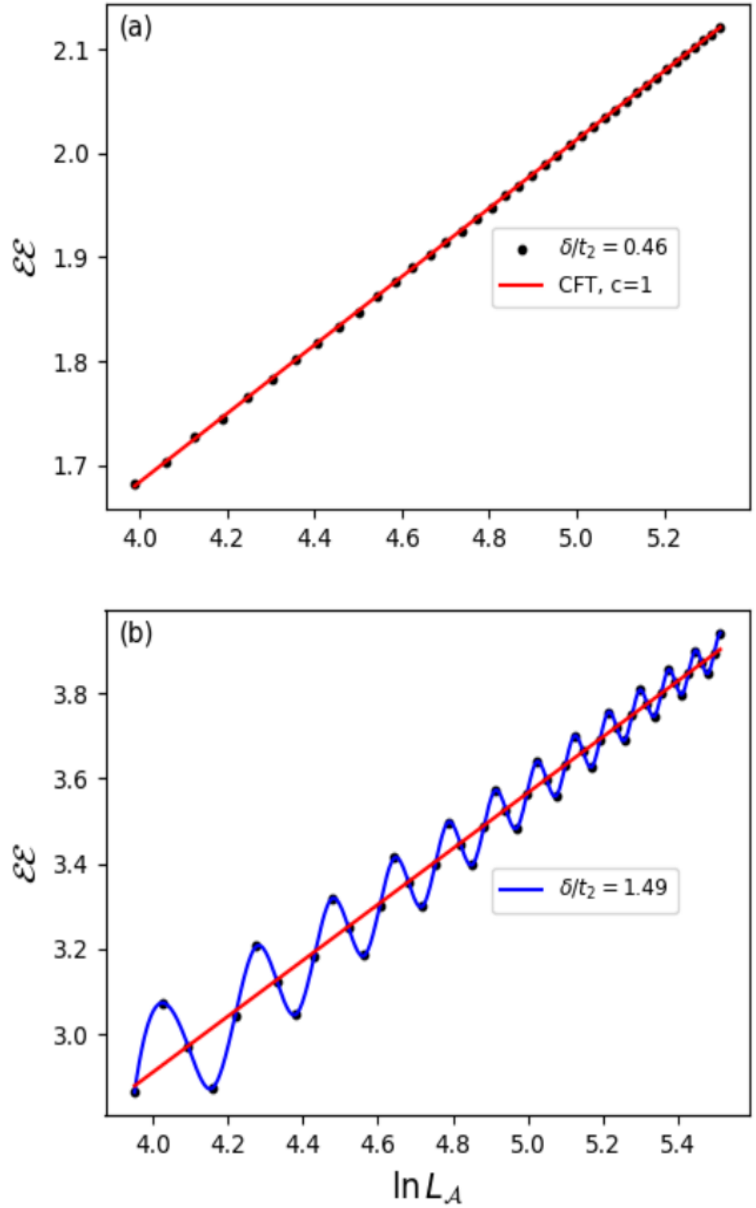
$L_{\mathcal{A}}$ subsystem size

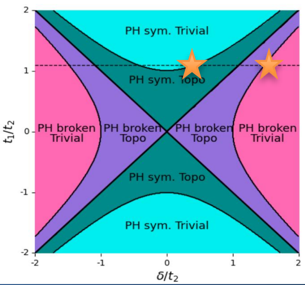
At $\delta/t_2 = 0.46$ linear $\mathcal{EE}$ vs $ln(L_{\mathcal{A}})$
Central charge c = 1

At $\delta/t_2 = 1.49$ oscillations are visible

Thermodynamic limit: oscillations vanish

Linear $\mathcal{EE}$ vs $ln(L_{\mathcal{A}})$. Central charge c=2?





Thermodynamics of the nH-SSH model around δ = 0.46

Thermal entropy of the edge:

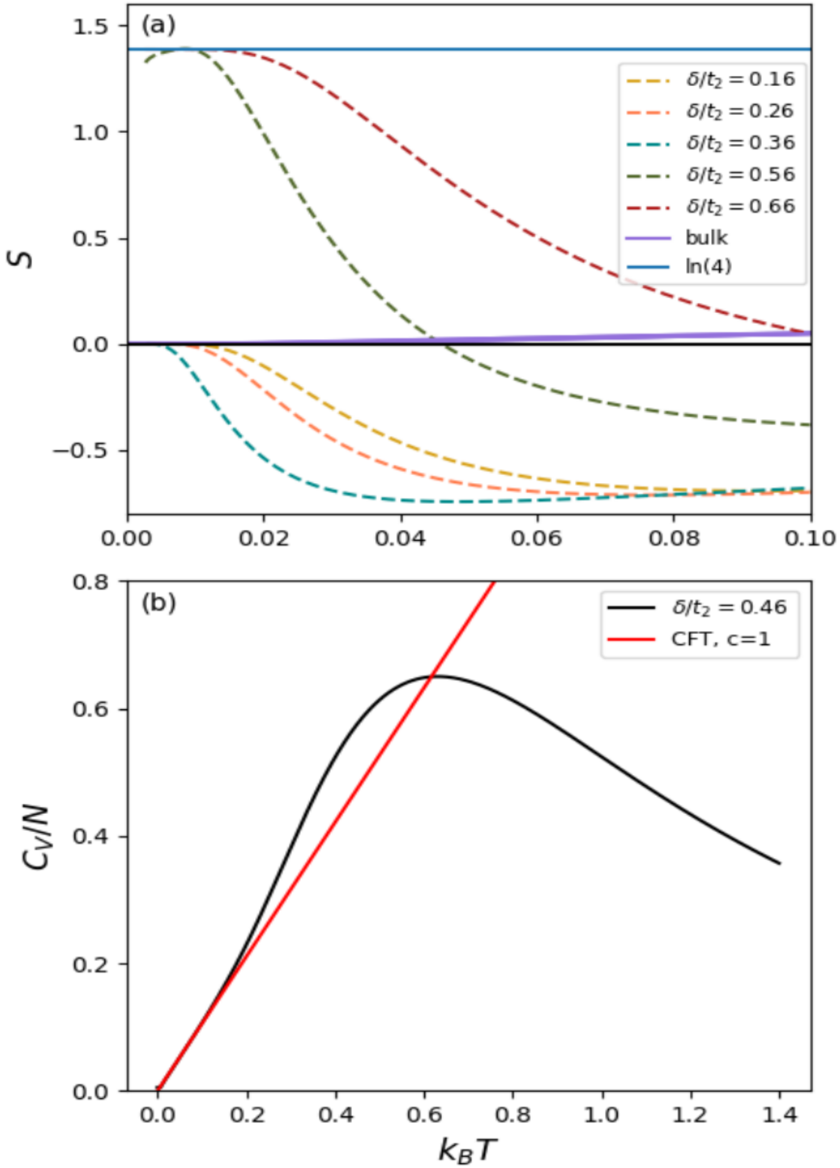
$$S = -\partial_T \Omega_{Edge} = -\partial_T(\Omega_{OBC} - \Omega_{sur})$$

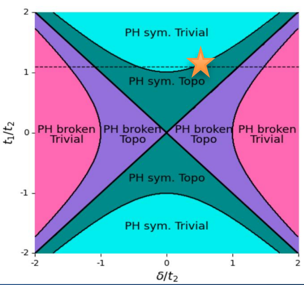
Trivial and topological phase: Bulk entropy starts at zero and grows smoothly

$\delta/t_2 < 0.46$: Edge entropy 0, then becomes negative increasing T

$\delta/t_2 > 0.46$: Edge entropy ln 4 for T = 0, decays to 0 as T>0

$\delta/t_2 = 0.46$: Linear behavior of Bulk heat capacity at low temperatures. Agreement with CFT description (central charge c = 1)





Thermodynamics of the nH-SSH model δ = 1.1

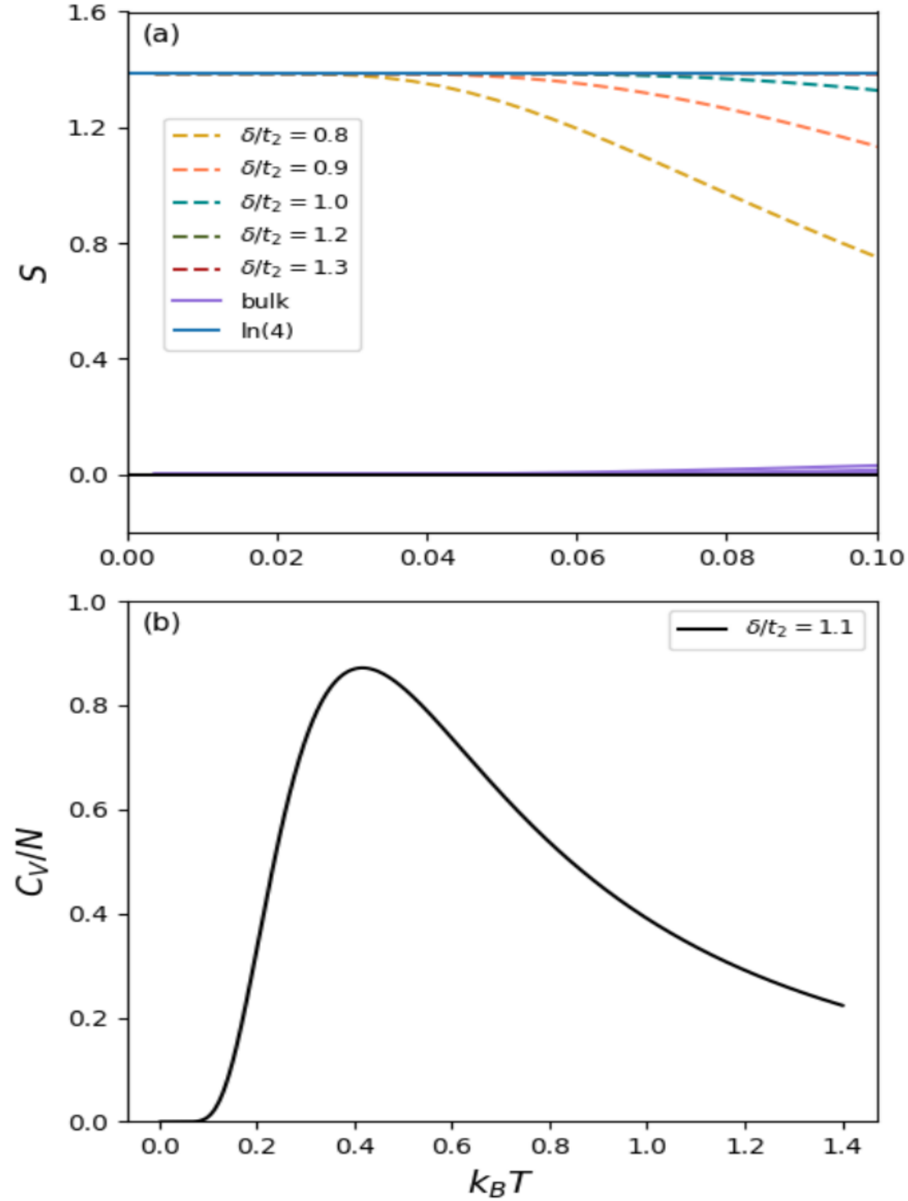
Non-Bloch band collapse

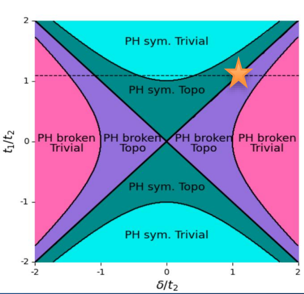
PH protected and PH broken (topo): Bulk entropy starts at zero and grows smoothly

$\delta/t_2 < 1.1$: PH-symmetric topological
Edge entropy ln 4, decays for T>0

$\delta/t_2 > 1.1$: PH-broken topological
Edge entropy ln 4, remains constant for T>0

At $\delta/t_2 = 1.1$: C_V no linear scaling at low T.
Remains zero until a critical T~0.1.





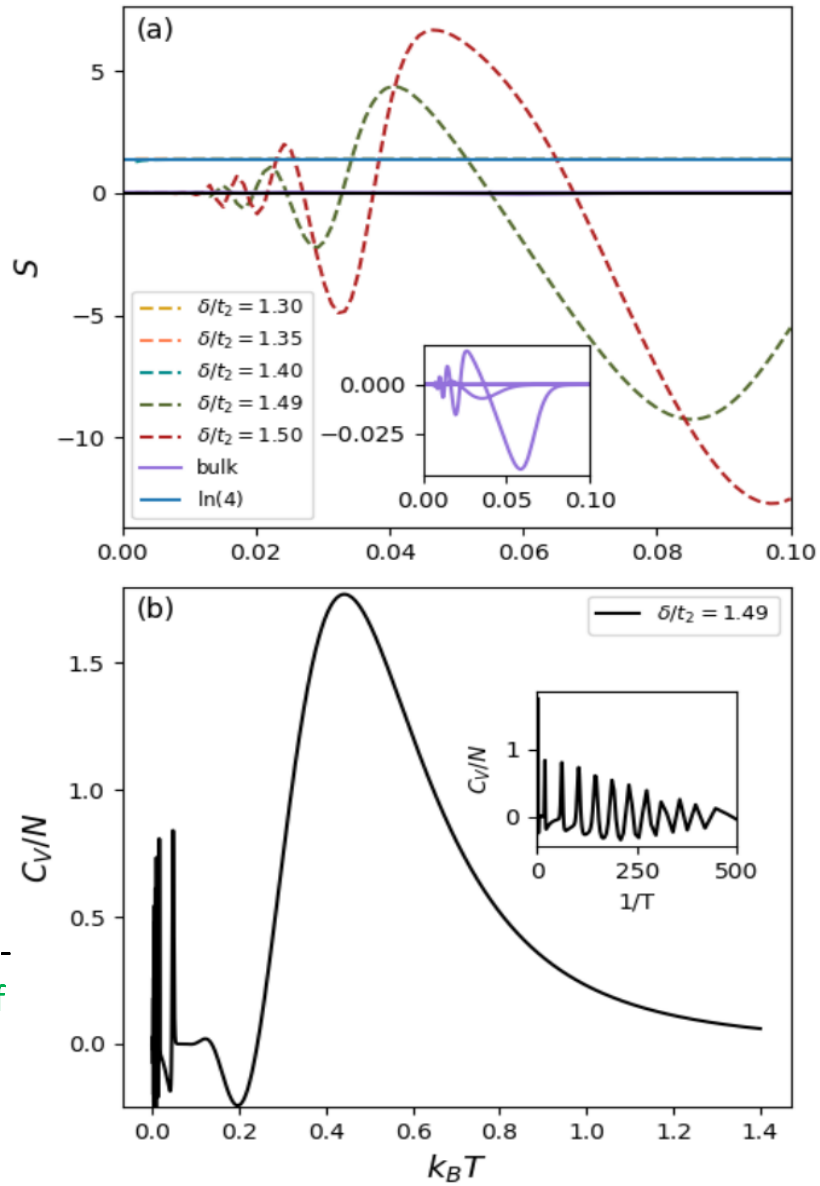
Thermodynamics of the nH-SSH model δ = 1.49

PH broken topological and trivial: Oscillations of bulk entropy in 1/T

$\delta/t_2 < 1.49$: Edge entropy at ln(4)
Constant for T>0

$\delta/t_2 > 1.49$: Edge entropy jumps to 0
T>0 oscillatory behavior.

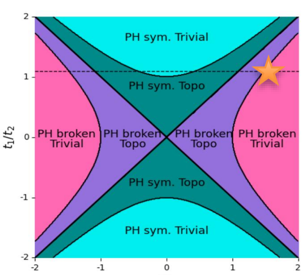
Bulk C_V: Oscillations periodic in 1/T
Characterization of an imaginary time crystal.



Resonance condition: $i\hbar\omega_n = E_{\mathrm{surr},\pm}(k)$

Period of peaks: $$\omega_n = (2n+1)\pi k_B T/\hbar$$

$$2\pi k_B/\sqrt{\delta^2 - t_1^2 - 1}$$



Oscillations in T spontaneous breaking of T-symmetry in imaginary time. Emergence of an imaginary time crystal

F. Wilczek, Phys. Rev. Lett. 109, 160401 (2012).
R. Arouca, E. C. Marino, and C. Morais Smith, Quantum Front. 1, 2 (2022).

Conclusions and outlook

We identified topological phase transitions for the non-reciprocal nH-SSH model at finite temperature

The entanglement entropy of the non-reciprocal nH-SSH model was studied to identify topological phases at zero temperature

The thermodynamics of the non-reciprocal nH-SSH model agrees with the results of the entanglement entropy for all phases

Heat capacity show that the model is a CFT fermionic system for some parameters

The entanglement entropy and the thermal entropy of the edge host fermionic zero modes in the PH-symmetry protected topological phase

At the nBBC critical point, the non-Hermitian SSH model is not a CFT

Oscillations were observed for both, the entanglement entropy and heat capacity. For the latter this means that there is the emergence of an “imaginary time crystal”

Thanks

Gracias

Dankjewel

Questions?